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Nonlinear interaction of an ultraintense electromagnetic wave and the self-created electron-positron plasma

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Abstract

The nonlinear interaction between the electron-positron pairs produced by an electromagnetic wave in a plasma is investigated for a circularly polarized wave using the relativistic Vlasov equation with a source term based on the Schwinger formula for the pair creation rate.

1 Introduction

The production of electron-positron pairs under the action of electromagnetic field (Schwinger effect) is a nonlinear effect that lies beyond the limits of perturbation theory.

The study of this effect can shed light on the nonlinear properties of the quantum electrodynamics (QED) vacuum.

This effect was first predicted for the case of a constant electric field more than 60 years ago

F. Sauter, Z. Phys. **98**, 714 (1931), **98**, 714 (1931);
W. Heisenberg and H.Z. Euler, Z. Phys. **98**, 714 (1936);
J. Schwinger, Phys. Rev. **82**, 664 (1951).

A plane electromagnetic wave in vacuum cannot produce electron-positron pairs: both its Lorentz invariants vanish

$$(E^2 - B^2)/2 = 0$$
$$\mathbf{E} \cdot \mathbf{B} = 0$$

For this reason this effect was first considered in the case of a constant electric field, in which case the first invariant $(E^2 - B^2)/2$ does not vanish.

Later, this analysis was extended to the case of a spatially homogeneous time-varying electric field

E. Brezin, C. Itzykson, Phys. Rev. D **2**, 1191 (1970);
V.S. Popov, JETP Lett. **13**, 185 (1971);
V.S. Popov, Sov. J. Nucl. Phys. **19**, 584 (1974);
M.S. Marinov, V.S. Popov, Sov. J. Nucl. Phys.**16**, 449 (1973);
N.B. Narozhny A.I. Nikishov, Sov. Phys. JETP **38**, 427 (1974).

These results were long believed to be of academic interest only, because the power of the laser systems available at that time was far below the limit for pair production to become experimentally observable.

However the recent development of laser technology has resulted in the increase of the power of optical and infrared lasers by many orders of magnitude. Presently, lasers systems are available that can deliver pulses with intensities of the order of 10^{22} W/cm^2 in the focal spot.

Such intensities are still much smaller than the characteristic intensity for pair production $I_{Sch} = 4.6 \times 10^{29}$ W/cm^2, which corresponds, for a laser pulse with wavelength $\approx 1\mu m$, to an electric field equal to the critical Schwinger field $E_{Sch} = 1.32 \times 10^{16}$ V/cm.

Nevertheless, there are projects that aim to reach intensities as high as $10^{26} - 10^{28}$ W/cm^2 already in the coming decade. In addition, several methods for reaching the critical intensity with presently available systems have been proposed recently.

One of these schemes was demonstrated in the experiments at SLAC where 10^{18} W/cm^2 laser photons, backscattered by a 46.6 GeV electron beam, interacted with the laser pulse and several electron-positron pairs were detected

D.L. Burke, *e*t al., Phys. Rev. Lett. **7**9, 1626 (1997).

Another scheme for reaching critical intensities was suggested by

S.V. Bulanov, *e*t al., Phys. Rev. Lett. **91**, 085001 (2003)
where the interaction of the laser pulse with electron density modulations in a plasma, produced by a counter-propagating breaking wake plasma wave, results in the frequency up-shift and pulse focusing.

In this scheme intensities of the order of the critical density can be obtained using 10^{18} W/cm^2 laser pulses.

Hence, a detailed study of the Schwinger effect in time-varying electromagnetic fields and of all the processes that accompany has become an interesting problem from an experimental point of view also.

The process of electron-positron pair production by electromagnetic fields for which $E^2 - B^2$ does not vanish and that are *solutions of the Maxwell equations in a plasma and in vacuum* was studied recently in

S. S. Bulanov, Phys. Rev. E. **69**, 036408 (2004).

S. S. Bulanov, N.B. Narozhny, V.D. Mur, V.S. Popov, Phys. Lett. A **330**, 1 (2004)

It was shown that already for intensities of the laser pulse smaller than the critical intensity, the energy loss due to pair production is of the same order of the energy storied in the pulse: *it is not possible to consider the electromagnetic field in the pulse as an external field and the energy loss by the electromagnetic field due to pair production and particle acceleration must be taken into account.*

We consider

1) the process of electron-positron pair production in a cold collisionless plasma, under the action of an electromagnetic field which is an actual solution of the Maxwell equations,

2) the backreaction of the produced pairs on the background field. In doing so we use the Boltzmann-Vlasov equation, with a source term obtained from the pair production rate.

S.S. Bulanov, A.M. Fedotov, F. Pegoraro, Phys Rev E, 71, 016404- 016404-11 (2005);
S.S. Bulanov, A.M. Fedotov, F. Pegoraro, JETP Letters 80, 734-738 (2004).

We consider a planar, circularly polarized, electromagnetic wave propagating in an underdense collisionless electron-positron plasma.

In the case of a plane wave in a plasma the first field invariant $(E^2 - B^2)/2$ is not zero.

Therefore, in a plasma, electron-positron pairs can be produced by a plane electromagnetic wave.

A Lorentz transformation to the reference frame moving with the group velocity v_g of the wave transforms the electromagnetic field into a purely electric field, that rotates with constant frequency, and with no associated magnetic field.

This transformation reduces the problem under consideration to the situation where the pairs are produced by a time-varying electric field.

We study the effect on the background wave in the plasma caused by the pairs produced by the wave through their polarization and conduction currents.

We consider the interaction between the wave and the plasma as an initial value problem in the moving frame and study the evolution of the wave electromagnetic field.

We find a strong nonlinear dependence of wave field properties on the wave initial amplitude.

We find a nonlinear up-shift of the wave frequency, a change of its polarization state and damping of its amplitude.

In the reference frame moving with the wave group velocity v_g the magnetic field of the wave vanishes and its time-varying electric field is spatially homogeneous and is thus governed by the equation

$$\frac{d\mathbf{E}}{dt} = -4\pi\mathbf{j} = -4\pi \sum_{\alpha=+,-} \frac{e_\alpha}{(2\pi)^3} \int \mathbf{v}_\alpha f_\alpha(\mathbf{p}, t) d^3p. \quad (1)$$

where

$$\mathbf{v} = \mathbf{p}/(m^2 + p^2)^{1/2},$$

$$p = |\mathbf{p}| = (p_x^2 + p_y^2 + p_z^2)^{1/2},$$

$f_\alpha(\mathbf{p}, t)$ is the positron (electron) distribution function, normalized such that

$$\int f_\alpha(\mathbf{p}, t) d^3\, p/(2\pi)^3 = n_\alpha$$

gives the number n_α of electrons or positrons per unit volume, and e_α is their electric charge with $\alpha = +$ for the positrons and $\alpha = -$ for the electrons.

The relativistic kinetic equation

$$\frac{\partial f_\alpha}{\partial t} + e_\alpha \mathbf{E} \frac{\partial f_\alpha}{\partial \mathbf{p}} = q_\alpha(E, p), \qquad (2)$$

describes the dependence on time and momentum of the distribution function $f_\alpha(\mathbf{p}, t)$ in the moving frame where a spatially homogeneous electric field $\mathbf{E}$ is present. The source term in Eq.(2) is proportional to the quasiclassical probability

$$\exp\left[-\frac{\pi(m^2 + p_\perp^2)}{|e\mathbf{E}(t)|}\right]. \qquad (3)$$

of tunneling through the gap between the lower and the upper continuum of electron energy spectrum in the presence of a time-varying electric field with time playing the role of a parameter.

We assume that the pairs are produced at rest,

$$q_\alpha(E, p) = 2e^2\mathbf{E}^2(t) \exp\left[-\frac{\pi m^2}{|e\mathbf{E}(t)|}\right] \delta\left(\mathbf{p}\right), \qquad (4)$$

where $\int q_\alpha(E, p) d^3\ p/(2\pi)^3$ gives Schwinger's formula

$$[(eE)^2/4\pi^3] \exp[-\pi m^2/eE],$$

We integrate Eq.(2) along the particle characteristics

$$p_{\parallel} = p_{\parallel,0}, \qquad \frac{dp_{\perp}}{dt} = e_{\alpha} E, \qquad \frac{df_{\alpha}}{dt} = q_{\alpha}. \tag{5}$$

Defining $\mathbf{A}(t) = -\int_0^t \mathbf{E} ds$, we obtain

$\mathbf{p}_{\perp} + e_{\alpha}\mathbf{A}(t) = \mathbf{p}_{\perp 0}$

and

$$f_{\alpha} = f_{\alpha,0}[p_{\parallel}, \mathbf{p}_{\perp} + e_{\alpha}\mathbf{A}(t)] + \int_0^t q_{\alpha}\{\mathbf{p}_{\perp} + e_{\alpha}[\mathbf{A}(t) - \mathbf{A}(t')], t'\} dt'.$$

Assume cold initial distribution

$$f_{\alpha,0} = n_0 (2\pi)^3 \delta(p_{\perp}) \delta(p_{\parallel} - p_{\parallel,0})$$

where $p_{\parallel,0}$ is the parallel momentum which arises from the Lorentz transformation from the laboratory to the moving frame.

The electron-positron pair production by a spatially homogeneous time-varying electric field generates a polarization current

$$\frac{d\mathbf{E}}{dt} = -4\pi \mathbf{j}_{tot} = -4\pi \left(\mathbf{j}_{cond} + \mathbf{j}_{pol} \right), \tag{6}$$

where the conduction current is

$$\mathbf{j}_{cond}(t) = e \sum_{\alpha=+,-} \int f_{\alpha}(\mathbf{p}, t) \frac{\mathbf{p}}{(m^2 + p^2)^{1/2}} \frac{d^3 p}{(2\pi)^3}, \tag{7}$$

and the polarization current is

$$\mathbf{j}_{pol}(t) = \frac{\mathbf{E}(t)}{|\mathbf{E}(t)|^2} \sum_{\alpha=+,-} \int q_{\alpha}(\mathbf{p}, t)(m^2 + p^2)^{1/2} \frac{d^3 p}{(2\pi)^3}. \tag{8}$$

Finally for the dimensionless vector-potential $\mathbf{a} = e\mathbf{A}/m$ and the normalized electric field $\mathbf{e} = e\mathbf{E}/m^2$, we obtain

$$\frac{d\mathbf{a}(t)}{dt} = -m\mathbf{e}(t), \tag{9}$$

$$m\frac{d\mathbf{e}(t)}{dt} = \omega_p^2 \frac{\mathbf{a}(t)}{[1 + \tilde{p}_{\|0}^2 + a^2(t)]^{1/2}} \tag{10}$$

$$+\frac{\kappa}{m}\int_0^t \frac{\mathbf{a}(t) - \mathbf{a}(t')}{[1 + |\mathbf{a}(t) - \mathbf{a}(t')|^2]^{1/2}} \frac{|\mathbf{e}(t')|^2}{8\pi^3} \exp\left[-\frac{\pi}{|\mathbf{e}(t')|}\right] dt'$$

$$-\frac{em^2}{2\pi^2}\mathbf{e}(t) \exp\left[-\frac{\pi}{|\mathbf{e}(t)|}\right],$$

where
ω_p is the non-relativistic Langmuir frequency,
$\tilde{p}_{\|0} \equiv p_{\|0}/m$
$\kappa = 8\pi e^2 m^4$, (the factor m^4 stands for the inverse of the invariant Compton 4-volume $m^4 = c/l_c^4 \approx 0.14 \times 10^{53}$ cm^{-3} s^{-1}).

The process of electron-positron pair production leads to the damping of the wave in the plasma and to the nonlinear up-shift of its frequency.

The damping is due to the fact that each pair creation takes a portion of the field energy equal to $2mc^2$ as well as the amount needed for the particle acceleration.

The up-shift of the field frequency is due to the increase of the plasma density, and thus of the Langmuir frequency, as new pairs are created.

This frequency up-shift is seen in Fig.1, and bears a strong resemblance to the blue-shift of an electromagnetic wave that propagates in a medium that becomes ionized.

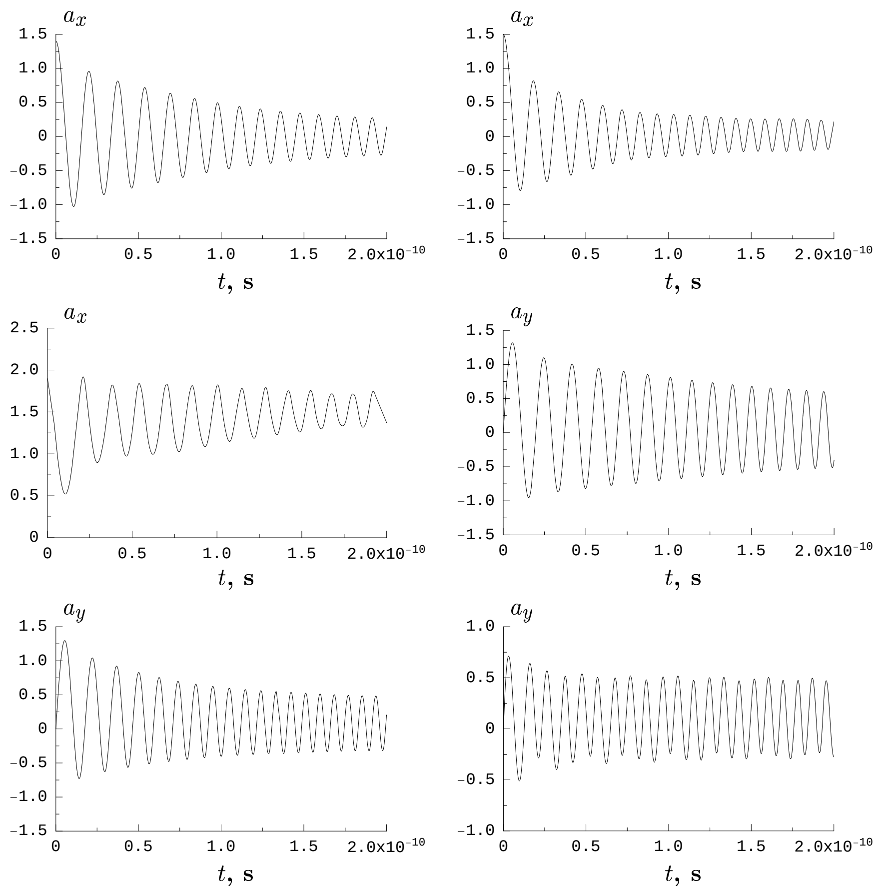
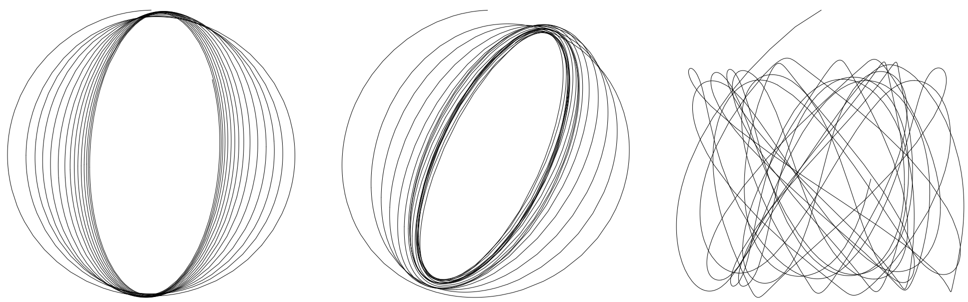


Figure 1: Time evolution in the moving frame of the x and the y-components of the dimensionless vector potential for different initial amplitudes: $a = 1.4 \times 10^5$ (a,d), $a = 1.5 \times 10^5$ (b,e), $a = 1.9 \times 10^5$ (c,f) with initial plasma density $n_0 = 10^{19}\ cm^{-3}$ in the moving frame; $v_g \approx 1$, $\gamma_g = 10$. The upper row shows the x-component of the vector-potential, the lower the y-component. On the x-axis time is measured in seconds; $a = 1$ corresponds, for a 1 μm wavelength pulse, to an intensity of 10^{18} W/cm^2 and $a = 4.6 \times 10^5$ to the Schwinger intensity.

Since the pair production rate depends on the field amplitude exponentially, an unbalanced damping of the field components can occur and lead to a change of the field polarization.

The decrease of the amplitude of the vector-potential is accompanied by a frequency up-shift, so that the decrease of the amplitude of the electric field is not as fast as that of the vector-potential.

These properties of the electric field are shown in Fig.2, where the projections of the polarization vector are presented for the same set of initial parameters as in Fig.1.



a) b) c)

Figure 2: Trajectories of the projections of the electric field polarization vector for the same set of initial conditions as in Fig.1.

In Fig.2a we see the damping of the x-component of the electric field and the transition from circular to elliptic polarization with the major axis of the ellipse directed along the y-axis.

In Fig.2b we see a rotation of the principal axes of the ellipse. The situation shown in Fig.2c is different from the two previous ones. In this latter case the pair production rate at the beginning of the field evolution is so large (see Fig. 3c) that the first wave oscillation cycle cannot be completed, leading to oscillations of the x-component of the wave vector potential around a non-zero mean value determined by the balance between the time averaged parts of the first two terms on the r.h.s. of the second of Eqs. (10).

Conclusions

Interesting phenomena of nonlinear plasma dynamics are at play in a laser produced electron positron plasma

Reaching such plasma regimes in laboratory plasmas may be not too far away.