

# **A Landau fluid model for dispersive magnetohydrodynamics**

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In spatial plasmas, collisions often negligible: usual MHD questionable.

For large-scale dynamics, hydrodynamic approach nevertheless advantageous.

Interest in constructing fluid models that extend MHD equations to collisionless situations by including finite Larmor radius (FLR) corrections and Landau damping (the only fluid-particle resonance that can affect large scales).

Most useful to analyze the dynamics of the magnetosheath (buffer between the earth bow shock and the magnetopause, that plays an important role: decrease the impact of solar activity on the earth environment).

Magnetosheath displays a **wide spectrum of low frequency modes** (Alfvén, slow and fast magnetosonic, mirror). Cluster spacecrafts allow one to determine  $k$ -spectra and clearly identify modes (Sahraoui et al. 2004).

Size of perturbations can be **smaller than the ion gyroradius**.

The plasma is relatively **warm and collisionless**.

**Landau damping and finite Larmor radius corrections** play an important role.

Evidence of **coherent solitonic structures** (magnetic holes and shocklets), whose origin is still debated (Tsurutani et al. 2004).

## Which tool?

- Description of intermediate-scale dynamics by usual MHD is questionable.
- Numerical integration of Vlasov-Maxwell or gyrokinetic equations often beyond the capabilities of present day computers.
- Need for a reduced description that retains most of the aspects of a FLUID MODEL but INCLUDES REALISTIC APPROXIMATIONS OF THE PRESSURE TENSOR AND WAVE-PARTICLE RESONANCES.

Should remain simple enough to allow 3D numerical simulations of turbulent regime.

- ★ Gyrofluids: hydrodynamic moments obtained from gyrokinetic equations. Capture high order FLR corrections but need a specific closure and are written in a local reference frame.
- ★ Landau fluids [Hammett and co-authors (1990s)]: monofluid taking into account wave-particle resonances in a way consistent with linear kinetic theory.

## Outline of the method

- **Goal:** Extend Landau-fluid model, to reproduce the weakly nonlinear dynamics of dispersive MHD (magnetosonic and Alfvén) waves whatever their direction of propagation, in particular of kinetic Alfvén waves (KAW) with  $k\rho_L \leq 1$ , by retaining FLR corrections and a generalized Ohm's law in addition to Landau damping.
- **Starting point:** Vlasov-Maxwell (VM) equations.
- **Small parameter:** ratio between the ion Larmor radius and the typical (smallest) wavelength. Field amplitudes also supposed to be small.
- **Main problem:** Exact hydrodynamic equations are obtained by taking moments of VM equations. The hierarchy must however be closed and the main work resides in a proper determination of the pressure tensor.
- **Assumptions:** Homogeneous equilibrium state with bi-Maxwellian distribution functions.

## The equations

From Vlasov-Maxwell equations, derive for each species  $r$  equations for

$$\text{density } \rho_r = m_r n_r \int f_r d^3v,$$

$$\text{velocity } u_r = \frac{\int v f_r d^3v}{\int f_r d^3v},$$

$$\text{pressure tensor } \mathbf{p}_r = m_r n_r \int (v - u_r) \otimes (v - u_r) f_r d^3v$$

$$\text{heat flux tensor } \mathbf{q}_r = m_r n_r \int (v - u_r) \otimes (v - u_r) \otimes (v - u_r) f_r d^3v,$$

from which a one-fluid description can possibly be obtained for

$$\text{plasma density } \rho = \sum_r \rho_r, \text{ velocity } u = \frac{1}{\rho} \sum_r \rho_r u_r, \text{ total pressure } \mathbf{p} = \sum_r \mathbf{p}_r.$$

But pressure equation involves heat fluxes, and so on: [closure problem](#).

Furthermore, at hydrodynamical scales, pressure and heat flux tensors include [both fast and slow components](#).

Basic tensors

$$\boldsymbol{\tau} = \hat{\mathbf{b}} \otimes \hat{\mathbf{b}} \quad \mathbf{n} = \mathbf{I} - \boldsymbol{\tau}$$

where  $\hat{\mathbf{b}} = \mathbf{b}/B_0$

For each particle species (drop  $r$  index for simplicity)

Pressure tensor  $\mathbf{p} = \mathbf{P} + \boldsymbol{\Pi}$  sum of a gyrotropic pressure  $\mathbf{P} = p_{\perp} \mathbf{n} + p_{\parallel} \boldsymbol{\tau}$  (with  $2p_{\perp} = \mathbf{p} : \mathbf{n}$  and  $p_{\parallel} = \mathbf{p} : \boldsymbol{\tau}$ ) and of a gyroviscosity tensor  $\boldsymbol{\Pi}$  that satisfies  $\boldsymbol{\Pi} : \mathbf{n} = 0$  and  $\boldsymbol{\Pi} : \boldsymbol{\tau} = 0$ .

Equations for the [gyrotropic pressure components](#),

$$\begin{aligned} \partial_t p_{\perp} + \nabla \cdot (u p_{\perp}) + p_{\perp} \nabla \cdot \mathbf{u} - p_{\perp} \hat{\mathbf{b}} \cdot \nabla u \cdot \hat{\mathbf{b}} + \frac{1}{2} (\text{tr} \nabla \cdot \mathbf{q} - \hat{\mathbf{b}} \cdot (\nabla \cdot \mathbf{q}) \cdot \hat{\mathbf{b}}) \\ + \frac{1}{2} (\text{tr} (\boldsymbol{\Pi} \cdot \nabla u)^S - (\boldsymbol{\Pi} \cdot \nabla u)^S : \boldsymbol{\tau} + \boldsymbol{\Pi} : \frac{d\boldsymbol{\tau}}{dt}) = 0 \end{aligned}$$

$$\partial_t p_{\parallel} + \nabla \cdot (u p_{\parallel}) + 2p_{\parallel} \hat{\mathbf{b}} \cdot \nabla u \cdot \hat{\mathbf{b}} + \hat{\mathbf{b}} \cdot (\nabla \cdot \mathbf{q}) \cdot \hat{\mathbf{b}} + (\boldsymbol{\Pi} \cdot \nabla u)^S : \boldsymbol{\tau} - \boldsymbol{\Pi} : \frac{d\boldsymbol{\tau}}{dt} = 0.$$

Similar decomposition of the heat flux tensor  $\mathbf{q} = \mathbf{S} + \boldsymbol{\sigma}$  with the conditions  $\sigma_{ijk}n_{jk} = 0$  and  $\sigma_{ijk}\tau_{jk} = 0$ . One has

$$S_{ijk} = \frac{1}{2}(S_i^\perp n_{jk} + S_j^\perp n_{ik} + S_k^\perp n_{ij} + S_l^\perp \tau_{li} n_{jk} + S_l^\perp \tau_{lj} n_{ik} + S_l^\perp \tau_{lk} n_{ij}) \\ + S_i^\parallel \tau_{jk} + S_j^\parallel \tau_{ik} + S_k^\parallel \tau_{ij} - \frac{2}{3}(S_l^\parallel \tau_{li} \tau_{jk} + S_l^\parallel \tau_{lj} \tau_{ik} + S_l^\parallel \tau_{lk} \tau_{ij})$$

which is characterized by the **parallel and transverse heat flux vectors**  $S^\parallel$  and  $S^\perp$  with components  $S_i^\parallel = q_{ijk}\tau_{jk}$  and  $2S_i^\perp = q_{ijk}n_{jk}$ .

$\boldsymbol{\sigma}$  involves either nonlinear contributions or linear contributions of order  $0(\frac{k^2}{\Omega^2}v_{th}^2)$ , and thus turns out to be negligible in the equations for gyroviscosity or heat flux tensors.

Since  $m_e/m_i \ll 1$ : **only non-gyrotropic corrections due to ions are retained.**

Weakly nonlinear regime: **nongyrotropic contributions  $\boldsymbol{\Pi}$ ,  $S_\perp^\perp$  and  $S_\perp^\parallel$  retained at the linear level only.**

Fourth order moment are written in the form

$$\begin{aligned} \rho r_{ijkl} = & P_{ij}P_{lk} + P_{ik}P_{jl} + P_{il}P_{jk} + P_{ij}\Pi_{lk} + P_{ik}\Pi_{jl} + P_{il}\Pi_{jk} \\ & + \Pi_{ij}P_{lk} + \Pi_{ik}P_{jl} + \Pi_{il}P_{jk} + \rho \tilde{r}_{ijkl}. \end{aligned}$$

with a **gyrotropic form** for the tensor  $\tilde{r}$ :

$$\begin{aligned} \tilde{r}_{ijkl} = & \frac{\tilde{r}_{\parallel\parallel\parallel}}{3} (\tau_{ij}\tau_{kl} + \tau_{ik}\tau_{jl} + \tau_{il}\tau_{jk}) + \tilde{r}_{\parallel\perp} (n_{ij}\tau_{kl} + n_{ik}\tau_{jl} + n_{il}\tau_{jk} \\ & + \tau_{ij}n_{kl} + \tau_{ik}n_{jl} + \tau_{il}n_{jk}) + \frac{\tilde{r}_{\perp\perp}}{2} (n_{ij}n_{kl} + n_{ik}n_{jl} + n_{il}n_{jk}) \end{aligned}$$

Introducing the scalar quantities  $r_{\parallel\parallel\parallel} = r_{ijkl}\tau_{ij}\tau_{kl}$ ,  $r_{\perp\parallel} = \frac{1}{2}r_{ijkl}n_{ij}\tau_{kl}$  and

$r_{\perp\perp} = \frac{1}{4}r_{ijkl}n_{ij}n_{kl}$  (and similar definitions for the tilde quantities),

$$\tilde{r}_{\parallel\parallel\parallel} = r_{\parallel\parallel\parallel} - 3\frac{p_{\parallel}^2}{\rho} \quad \tilde{r}_{\parallel\perp} = r_{\parallel\perp} - \frac{p_{\perp}p_{\parallel}}{\rho} \quad \tilde{r}_{\perp\perp} = r_{\perp\perp} - 2\frac{p_{\perp}^2}{\rho}$$

$$\nabla_{\perp} \cdot \Pi_{\perp} = \frac{p_{\perp}^{(0)}}{2\Omega} \hat{z} \times \Delta_{\perp} u - \frac{\rho^{(0)}}{2m} r_L^2 \Delta_{\perp} \nabla_{\perp} T_{\perp}^{(1)} - \frac{1}{2\Omega^2} \Delta_{\perp} \nabla_{\perp} \tilde{r}_{\perp\perp} + \frac{1}{2\Omega} \hat{z} \times \partial_t \nabla_{\perp} \cdot \Pi_{\perp}$$

$$S_{\perp}^{\perp} = \frac{2p_{\perp}^{(0)}}{m\Omega} \hat{z} \times \nabla_{\perp} T_{\perp}^{(1)} - \frac{p_{\perp}^{(0)}}{2} r_L^2 \Delta_{\perp} u_{\perp} + \frac{2}{\Omega} \hat{z} \times \nabla_{\perp} \tilde{r}_{\perp\perp} + \frac{1}{\Omega} \hat{z} \times \partial_t S_{\perp}^{\perp}$$

$$\Pi_z = \frac{\hat{z}}{\Omega} \times (\nabla_{\perp} S_z^{\perp} + p_{\perp}^{(0)} \nabla_{\perp} u_z + p_{\parallel}^{(0)} \partial_z u_{\perp} - (p_{\perp}^{(0)} - p_{\parallel}^{(0)}) \partial_t \hat{b}_{\perp} + \partial_t \Pi_z)$$

$$- \frac{1}{\Omega^2} \partial_z \left( \frac{p_{\perp}^{(0)}}{m} \nabla_{\perp} T_{\parallel}^{(1)} - 2 \frac{p_{\perp}^{(0)} - p_{\parallel}^{(0)}}{m} T_{\parallel}^{(0)} \partial_z \hat{b}_{\perp} + \nabla_{\perp} \tilde{r}_{\parallel\perp} \right)$$

$$S_{\perp}^{\parallel} = \frac{\hat{z}}{\Omega} \times \left( \frac{p_{\perp}^{(0)}}{m} \nabla_{\perp} T_{\parallel}^{(1)} - 2 \frac{p_{\perp}^{(0)} - p_{\parallel}^{(0)}}{m} T_{\parallel}^{(0)} \partial_z \hat{b}_{\perp} + \nabla_{\perp} \tilde{r}_{\parallel\perp} + \partial_t S_{\perp}^{\parallel} \right)$$

$$- \frac{2T_{\parallel}^{(0)}}{m\Omega^2} \partial_z (\nabla_{\perp} S_z^{\perp} + p_{\perp}^{(0)} \nabla_{\perp} u_z + p_{\parallel}^{(0)} \partial_z u_{\perp} - (p_{\perp}^{(0)} - p_{\parallel}^{(0)}) \partial_t \hat{b}_{\perp}).$$

$$\nabla_{\perp} \cdot \Pi_{\perp} = (\partial_x \Pi_{xx} + \partial_y \Pi_{xy}, \partial_x \Pi_{xy} + \partial_y \Pi_{yy}, 0),$$

$$\Pi_z = (\Pi_{xz}, \Pi_{yz}, \Pi_{zz} = 0).$$

$\tilde{r}_{\perp\perp}$  turns out to be negligible at large scale

Nonlinear equations for the longitudinal components of parallel and transverse heat flux vectors (retaining only lowest order nonlinearities)

$$\begin{aligned} & \partial_t S_z^{\parallel} + \nabla \cdot (S_z^{\parallel} u) + 3S_z^{\parallel} \partial_z u_z + 3p_{\parallel} (\hat{b} \cdot \nabla) \left( \frac{p_{\parallel}}{\rho} \right) - p_{\perp}^{(0)} \hat{b}_{\perp} \cdot \nabla_{\perp} \left( \frac{p_{\parallel}}{\rho} \right) \\ & + \frac{2p_{\parallel}^{(0)}}{\rho^{(0)}} (p_{\parallel}^{(0)} - p_{\perp}^{(0)}) \partial_z \hat{b}_z + \nabla \cdot (\tilde{r}_{\parallel\parallel} \hat{b}) - 3\tilde{r}_{\parallel\perp} \nabla \cdot \hat{b} - (b_{\perp} \cdot \nabla_{\perp}) \tilde{r}_{\parallel\perp} = 0, \end{aligned}$$

$$\begin{aligned} & \partial_t S_z^{\perp} + \nabla \cdot (u S_z^{\perp}) + S_z \nabla \cdot u + p_{\parallel} (\hat{b} \cdot \nabla) \left( \frac{p_{\perp}}{\rho} \right) - 2p_{\perp}^{(0)} (\hat{b}_{\perp} \cdot \nabla_{\perp}) \left( \frac{p_{\perp}}{\rho} \right) \\ & + \frac{p_{\perp}^{(0)}}{\rho^{(0)}} (\partial_x \Pi_{xz} + \partial_y \Pi_{yz}) + \nabla \cdot (\tilde{r}_{\parallel\perp} \hat{b}) \\ & + \left( \frac{p_{\perp} (p_{\parallel} - p_{\perp})}{\rho} - \tilde{r}_{\perp\perp} + \tilde{r}_{\parallel\perp} \right) (\nabla \cdot \hat{b}) - (\hat{b}_{\perp} \cdot \nabla_{\perp}) \tilde{r}_{\perp\perp} = 0. \end{aligned}$$

## Fourth order moment closure

Turn to kinetic theory. Compute various hydrodynamic quantities using linearly perturbed distribution function, at second order in  $\omega/\Omega$ .

When comparing  $\tilde{r}_{\parallel\parallel\parallel}$  with  $S_z^{\parallel}$  or  $T_{\parallel}^{(1)}$ , one gets ( $\zeta = \frac{k_{\parallel}}{\Omega} \sqrt{\frac{m}{2T_{\parallel}^{(0)}}}$ ).

$$\tilde{r}_{\parallel\parallel\parallel} = \sqrt{\frac{2T_{\parallel}^{(0)}}{m}} \frac{2\zeta^2(1 + 2\zeta^2 R(\zeta)) + 3(R(\zeta) - 1) - 12\zeta^2 R(\zeta)}{2\zeta(1 - 3R(\zeta) + 2\zeta^2 R(\zeta))} S_z^{\parallel}.$$

$$\tilde{r}_{\parallel\parallel\parallel} = \frac{p_{\parallel}^{(0)} T_{\parallel}^{(0)}}{m} \frac{2\zeta^2(1 + 2\zeta^2 R(\zeta)) + 3(R(\zeta) - 1) - 12\zeta^2 R(\zeta)}{1 - R(\zeta) + 2\zeta^2 R(\zeta)} \frac{T_{\parallel}^{(1)}}{T_{\parallel}^{(0)}}.$$

Proceeding as in Snyder et al. (1997) , we write

$$\tilde{r}_{\parallel\parallel\parallel} = \beta_{\parallel} p_{\parallel}^{(0)} \frac{T_{\parallel}^{(0)}}{m} \left[ \frac{T_{\parallel}^{(1)}}{T_{\parallel}^{(0)}} - D_{\parallel} \sqrt{\frac{2T_{\parallel}^{(0)}}{m}} i \frac{k_z}{|k_z|} S_z^{\parallel} \right],$$

where  $\beta_{\parallel} = \frac{32-9\pi}{3\pi-8}$  and  $D_{\parallel} = \frac{2\sqrt{\pi}}{3\pi-8}$  are determined by matching with the exact kinetic expressions in the isothermal  $|\zeta| \ll 1$  and adiabatic limits  $|\zeta| \gg 1$ .

$\tilde{r}_{\parallel\perp}$  can be expressed in terms of  $S_z^\perp$  and the parallel current  $j_z$ .

$$\tilde{r}_{\parallel\perp} = \sqrt{\frac{2T_{\parallel}^{(0)}}{m} \frac{1 - R(\zeta) + 2\zeta^2 R(\zeta)}{2\zeta R(\zeta)}} [S_z^\perp + (b\Gamma_0(b) - b\Gamma_1(b))p_{\parallel}^{(0)} \left(\frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1\right) \frac{\Omega}{k_{\perp} c k_{\perp} B_0} \frac{4\pi j_z}{B_0}],$$

or using a two-pole Padé approximant for  $R$ , ( $D_{\perp} = \sqrt{\frac{\pi}{2}}$ )

$$\tilde{r}_{\parallel\perp} = -D_{\perp} \sqrt{\frac{2T_{\parallel}^{(0)}}{m} \frac{ik_z}{|k_z|}} [S_z^\perp + (b\Gamma_0(b) - b\Gamma_1(b))p_{\parallel}^{(0)} A \frac{\Omega}{k_{\perp} c k_{\perp} B_0} \frac{4\pi j_z}{B_0}]$$

$$\text{with } b = \frac{k_{\perp}}{\Omega} \sqrt{\frac{m}{2T_{\perp}^{(0)}}}, \quad \Gamma_n(b) = e^{-b} I_n(b) \approx \left(\frac{b}{2}\right)^n \text{ as } b \rightarrow 0,$$

$$A = \frac{T_{\perp}^{(0)}}{T_{\parallel}^{(0)}} - 1,$$

$\tilde{r}_{\perp\perp}$  negligible in the large scale limit.

## Validation

- For parallel Alfvén waves,  
Leading order (proton) gyroviscous tensor is sufficient

$$\begin{aligned}\pi_{xx}^{(1)} = -\pi_{yy}^{(1)} &= -\frac{p_{\perp}}{2\Omega}(\partial_y u_x + \partial_x u_y), & \pi_{zz}^{(1)} &= 0, & \pi_{xy}^{(1)} &= -\frac{p_{\perp}}{2\Omega}(\partial_y u_y - \partial_x u_x) \\ \pi_{yz}^{(1)} &= \frac{1}{\Omega}[2p_{\parallel}\partial_z u_x + p_{\perp}(\partial_x u_z - \partial_z u_x)], & \pi_{xz}^{(1)} &= -\frac{1}{\Omega}[2p_{\parallel}\partial_z u_y + p_{\perp}(\partial_y u_z - \partial_z u_y)]\end{aligned}$$

Only the longitudinal components  $S_z^{\perp}$  and  $S_z^{\parallel}$  of the transverse and parallel heat transfer vectors are relevant.

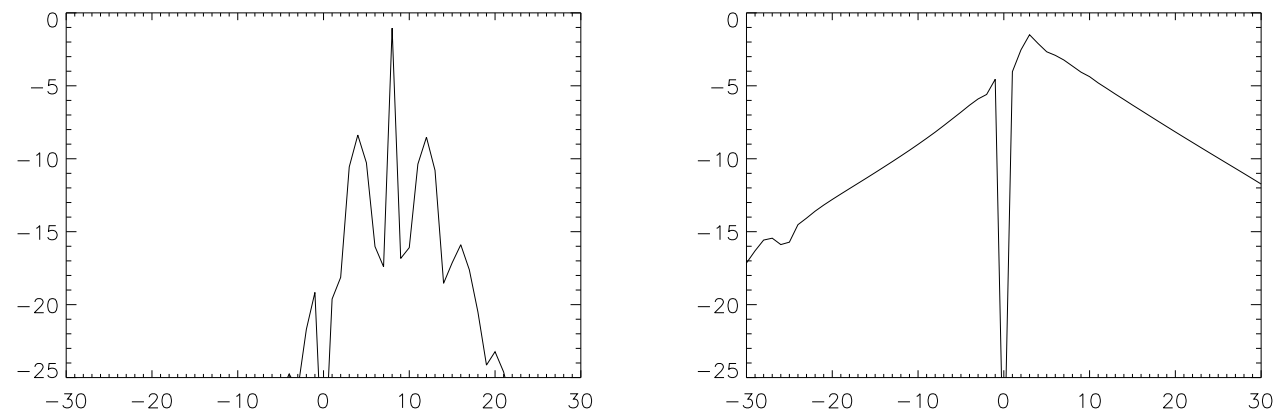
The long-wave reductive perturbative expansion performed on the resulting Landau-fluid model reproduces the **KDNLS equation** derived from Vlasov-Maxwell, up to the replacement of the plasma response functions by the corresponding two- or four-pole approximants.

**Consequence:** modulational type instabilities (including filamentation) of Alfvén waves and their weakly nonlinear developments are correctly reproduced.

Example:

For the left-handed mode at  $\beta = 1.5$  kinetic effects induce a **modulational instability** (Mjølhus & Wyller 1988) whereas fluid theory predicts only beat instability.

This is **verified numerically**, choosing  $k_0 = 0.408$  (length unit being the ion inertial length) and a pump amplitude  $B_0 = 0.3$ . **Ions significantly heated but not electrons.**



**Figure 1:** Spectral density for the complex quantity  $b_+$  in the linear phase of the modulational instability at  $t = 2000$  (left) and in the nonlinear phase at  $t = 3700$  (right) for the run with LH wave with  $B_0 = 0.3$ ,  $\beta = 1.5$  and  $T_e = 2T_p$ .

## DECAY INSTABILITY:

produces a forward propagating acoustic wave and a backward Alfvén wave with a wavenumber smaller than that of the pump.

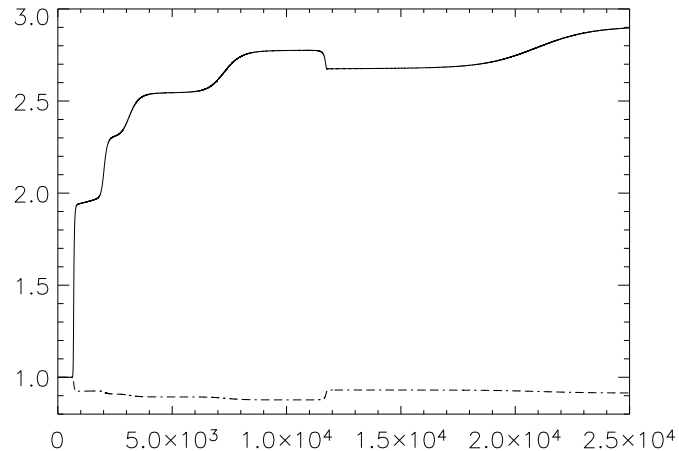
For the **right-handed mode** at  $\beta = .4$  and a pump amplitude  $b_0 = 0.1$ ,

an **algebraic inverse cascade** develops: EXCITATION IS TRANSFERED TO LARGER AND LARGER SCALES WHILE THE DIRECTION OF PROPAGATION OF THE WAVE SWITCHES ALTERNATIVELY AT EACH STEP OF THE PROCESS.

$\beta = 0.45$ , pump  $b_0 = 0.5$  and  $T_e = 0$ :

Each step is associated with a **parallel ion temperature increase**.

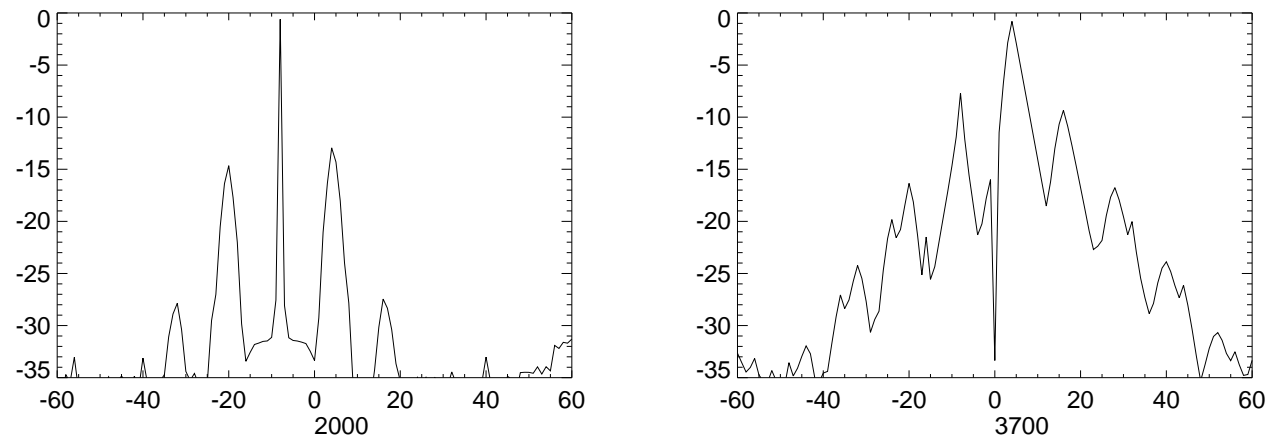
**Electrons remain cold**, which supports the isothermal fluid description of electrons in the hybrid code (Vasquez 1995).



**Figure 2:** Ion temperature evolution for a run with a right-handed wave with amplitude  $b_0 = 0.5$ , in a plasma with  $\beta = 0.45$  and  $T_e = 0$ .

Decay instability can persist at high values of  $\beta$ .

Taking  $R_p = \frac{v_A}{\Omega_p L_0} = 0.1$  (ratio of proton inertial length scale to reference length scale),  $b_0 = 0.5$  and  $\beta = 5$ , a decay instability is visible at early time, that leads to the dominance of the (backward)  $m = 4$  mode, whereas the fluid theory predicts a modulational instability.



**Figure 3:** Spectral density for the complex quantity  $b_+ = b_x + ib_y$  in the linear phase of the decay instability at  $t = 2000$  (left) and in the nonlinear phase at  $t = 3700$  (right) for the run with a right-hand polarized wave with amplitude  $b_0 = 0.5$ , in a plasma with  $\beta = 5$ ,  $R_p = 0.1$  and  $T_i/T_e = 1.5$ .

- Alfvén waves at finite angle of propagation:

FLR corrections of order  $1/\Omega_p^2$  are to be retained.

The governing equation is linear and reads, assuming  $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$ , (adiabatic protons and isothermal electrons) and  $\beta \ll 1$  ( $\xi$ : stretched coordinate along the propagation)

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2} \left[ \frac{\cos^3 \alpha}{\sin^2 \alpha} + \sqrt{\beta} \sqrt{\frac{\pi}{2}} \sqrt{\frac{m_e}{m_p}} \cos^3 \alpha \left( \tan^2 \alpha + \frac{1}{\tan^2 \alpha} \right) \mathcal{H} \right] \partial_{\xi\xi\xi} \frac{b_y}{B_0} = 0,$$

- Kinetic Alfvén waves ( $\cos^2 \alpha \ll \beta$ ):

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2} \cos \alpha \left[ -\beta \left( 1 + \frac{3T_p^{(0)}}{4T_e^{(0)}} \right) + \sqrt{\beta} \sqrt{\frac{\pi}{2}} \sqrt{\frac{m_e}{m_p}} \mathcal{H} \right] \partial_{\xi\xi\xi} \frac{b_y}{B_0} = 0.$$

agreement with Akhiezer et al. '75, Hasegawa and Chen '76.

- Magnetosonic waves:

Landau damping rate is (assuming  $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$ )

$$\gamma = -\sqrt{\beta} \sqrt{\frac{\pi}{8}} \sqrt{\frac{m_e \sin^2 \alpha}{m_p \cos \alpha}} \frac{(\omega^2 - \beta \cos^2 \alpha)^2 + \beta^2 \cos^4 \alpha}{(2\omega^2 - \beta - 1)(\omega^2 - \beta \cos^2 \alpha)} k v_A,$$

an expression identical to that found by a direct derivation from the Vlasov-Maxwell equations.

The long-wave equation is KdV+damping term.

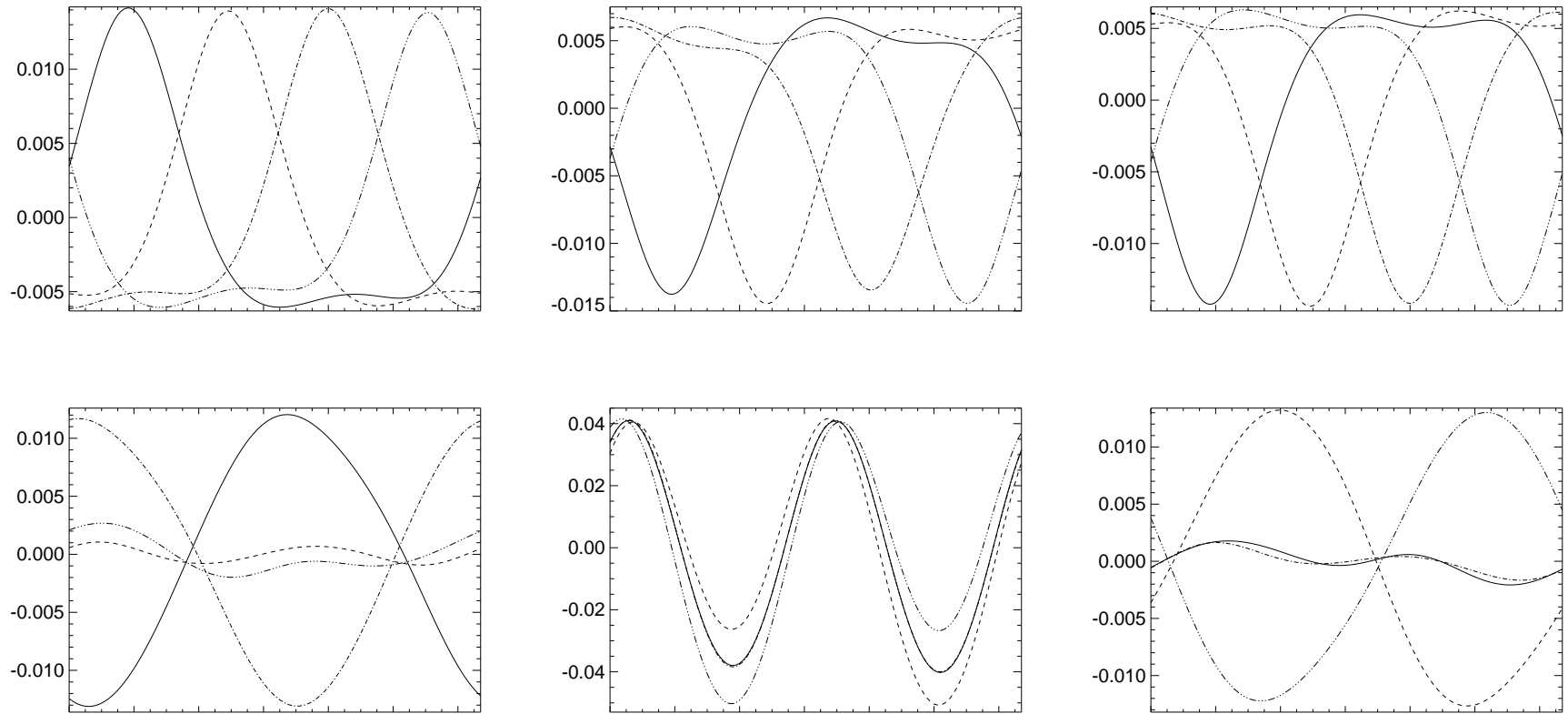


Figure 4: Profiles of the  $\tilde{u}_z$  component (left), of the density perturbation (middle) and of the magnetic field perturbation (right) at successive times (denoted by solid, dashed, dotted-dashed and triple-dotted-dashed lines) separated by 1.5 units starting at  $t = 104.3$  (top) and at  $t = 1502.3$  (bottom), in the case  $a_0 = 10^{-2}$ ,  $R_p = 10^{-1}$  and  $\beta = 10^{-3}$ .

Formation of SOLITONIC STRUCTURES with a hump for the parallel velocity and density depressions correlated with magnetic holes.

Eventually, the profile evolves to a quasi-stationary wave.

Dissipation of the magnetosonic wave very weak and Alfvén waves subdominant.

# Larger amplitude regime

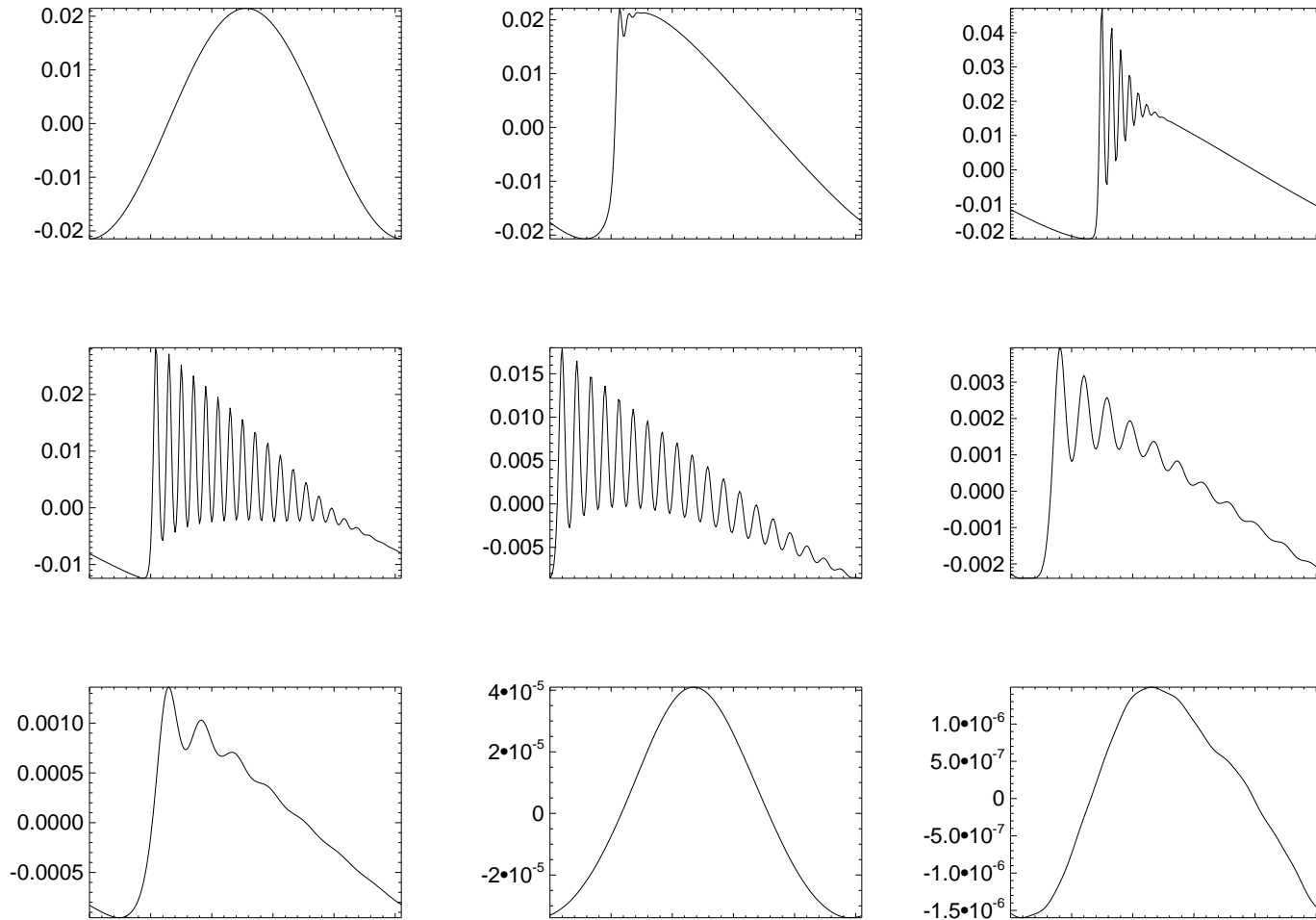


Figure 5: Snapshots at times  $t = 1, 32, 45, 101, 150, 501, 1002, 4000$  and  $17000$  of the longitudinal velocity component  $\tilde{u}_z$  for a magnetosonic wave of initial amplitude  $a_0 = 2.2 \cdot 10^{-2}$ , with  $\beta = 10^{-2}$  and  $R_p = 7 \cdot 10^{-3}$ .

Formation and dissipation of the high frequency modes in a **dispersive shocks**. Magnetosonic contribution is **completely dissipated**. Increase of  $T_{\parallel e}$  by 20%.

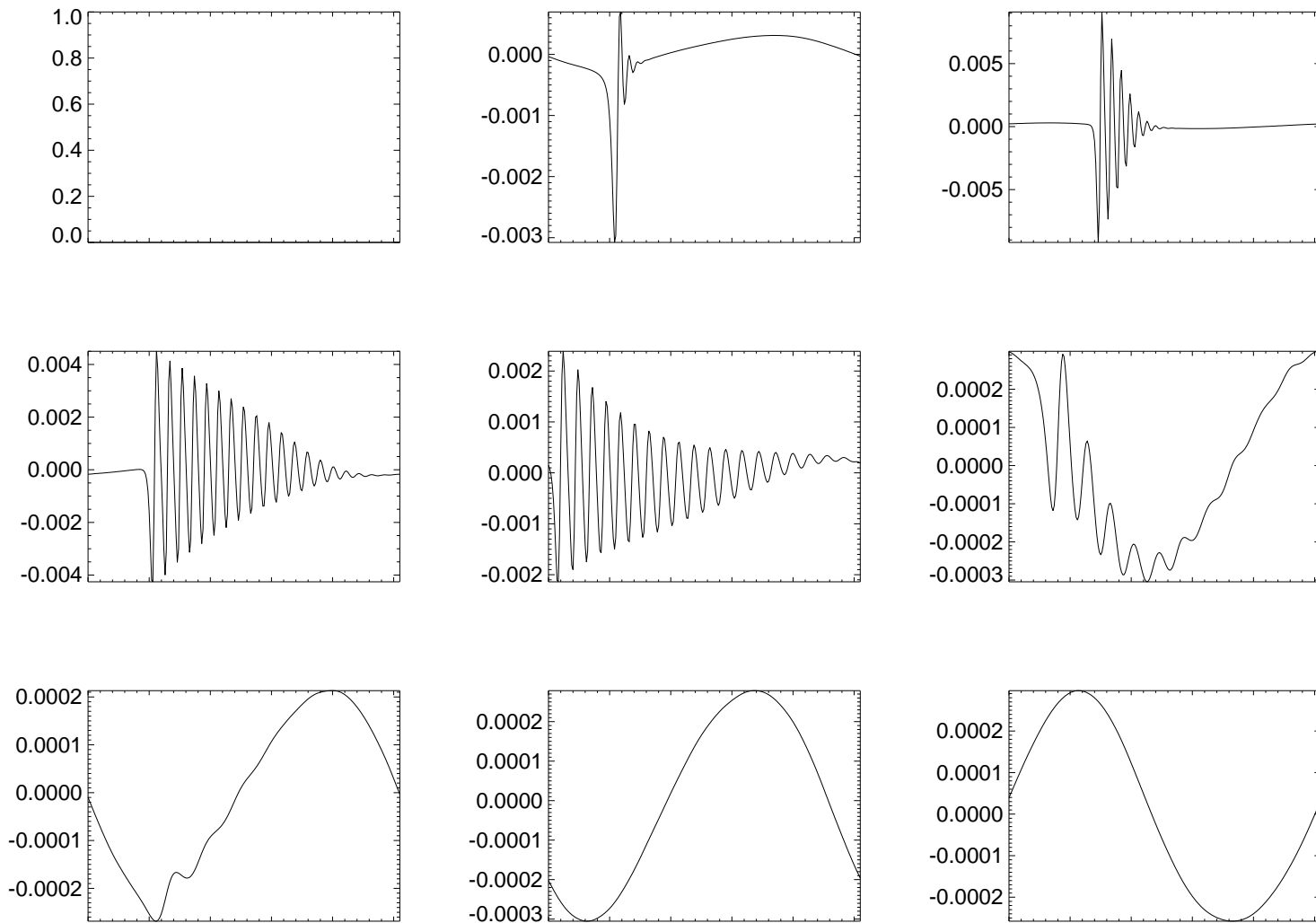


Figure 6: Same as before for the **transverse velocity component  $\tilde{u}_y$**  .

An **Alfvén wave** is generated that evolves to a **large-scale profile**. Small-scale oscillations being damped at a rate that scales like  $k^3/R_p^2$ . Resulting state: an **Alfvén wave essentially insensitive to Landau damping**.

- Transverse magnetosonic waves: (isotropic equilibrium temperatures)

Dispersion relation:

$$\omega^2 = k^2 v_A^2 (1 + \beta) \left( 1 - \frac{1}{4} k_{\perp}^2 r_L^2 \frac{1 + \beta_e + 5/2\beta_i}{1 + \beta_e + \beta_i} \right)$$

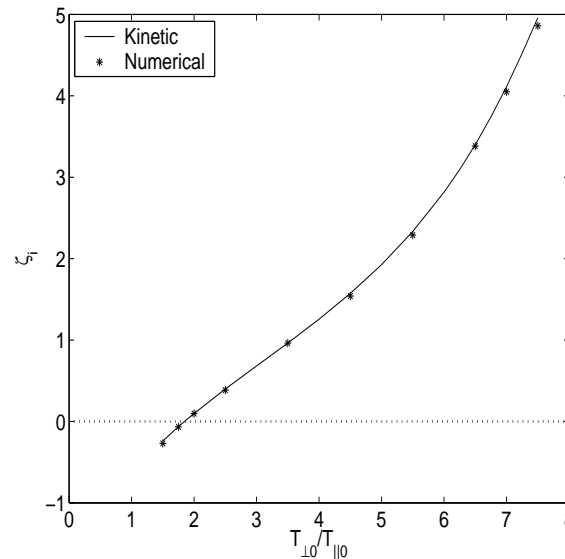
with  $\beta_r = \frac{8\pi p_r^{(0)}}{B_0^2}$ ,  $\beta = \beta_i + \beta_e$

In agreement with kinetic theory (Mikhailovskii and Smolyakov 1985)

**MIRROR MODES:** When  $T_{\perp} > T_{\parallel}$  mirror modes may become unstable.

The description of these modes requires an accurate modeling of the kinetic effects

When  $T_e = T_i$ , the condition for the instability reads  $\frac{p_{\perp}^{(0)}}{p_{\parallel}^{(0)}} > 1 + \frac{1}{8\pi} \frac{B_0^2}{p_{\perp}^{(0)}}$ . Assuming an angle of propagation such that  $\cos \alpha = 0.01$  and  $\beta = 1$  with  $R_p = 10^{-10}$  we numerically calculate the growth rate. **Excellent agreement with kinetic theory.**



**Figure 7:** Mirror mode growth rate predicted by the kinetic theory and given by time integration of the Landau fluid model, versus the equilibrium temperature anisotropy for a plasma with  $\beta = 1$  and equal temperatures for electrons and protons.

When Landau-fluid simulations are performed in situations where temperature anisotropy can excite mirror modes, the **smallest scales present in the code are linearly unstable** (at a rate proportional to their wavenumber).

**FLR corrections at SMALL transverse scales should be retained to stabilize the small scales** (comparable to the ion Larmor radius).

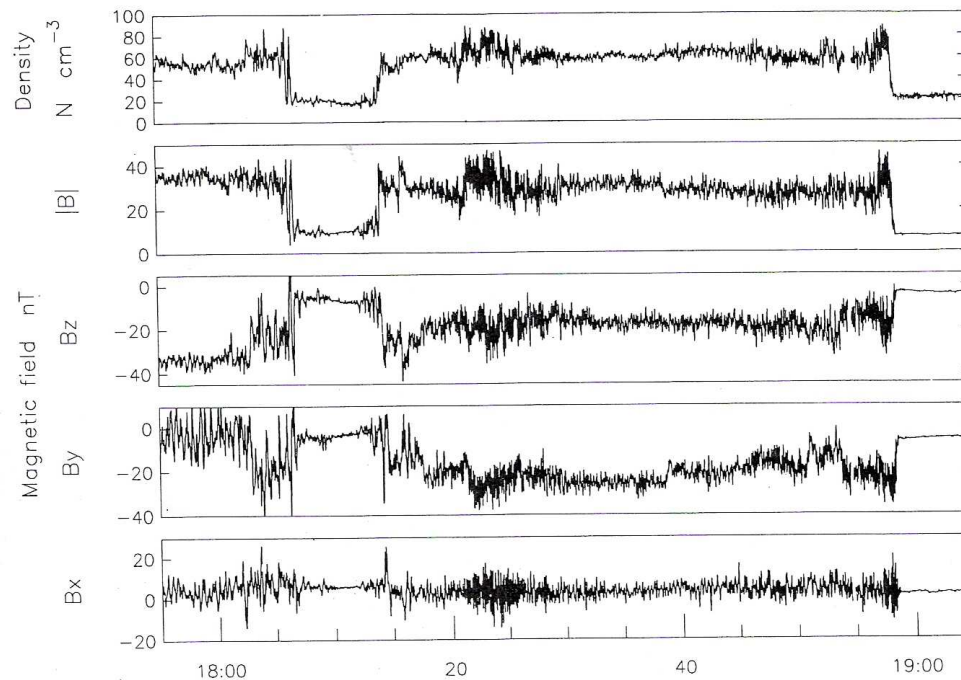


Figure 1. Density and magnetic field data (ISEE 1-2)

Figure 8: Mirror mode as measured by ISEE 1-2 spacecrafts (from Lacombe et al. 1992)

Mirror modes display nonlinear structures that were observed in the downstream perpendicular terrestrial bow shock.

One may expect that the hydrodynamic nonlinearities are sufficient to reproduce these structures, with FLR corrections prescribed by the linear kinetic theory.

A **promising approach** consists in expressing at the level of the LINEAR KINETIC THEORY, nongyrotropic contributions that are asymptotically slaved to the hydrodynamical variables, in terms of fluid variables in a form suitable for being incorporated into fluid equations: ELIMINATE EXPLICIT REFERENCE TO PLASMA RESPONSE FUNCTION.

The problem is to construct such a model that would be simple enough to permit accurate numerical simulations. Work in progress.

**Preliminary model:** the fluid hierarchy is closed at the level of the pressure, for a quasi-isothermal plasma, retaining FLR corrections at small transverse scales.

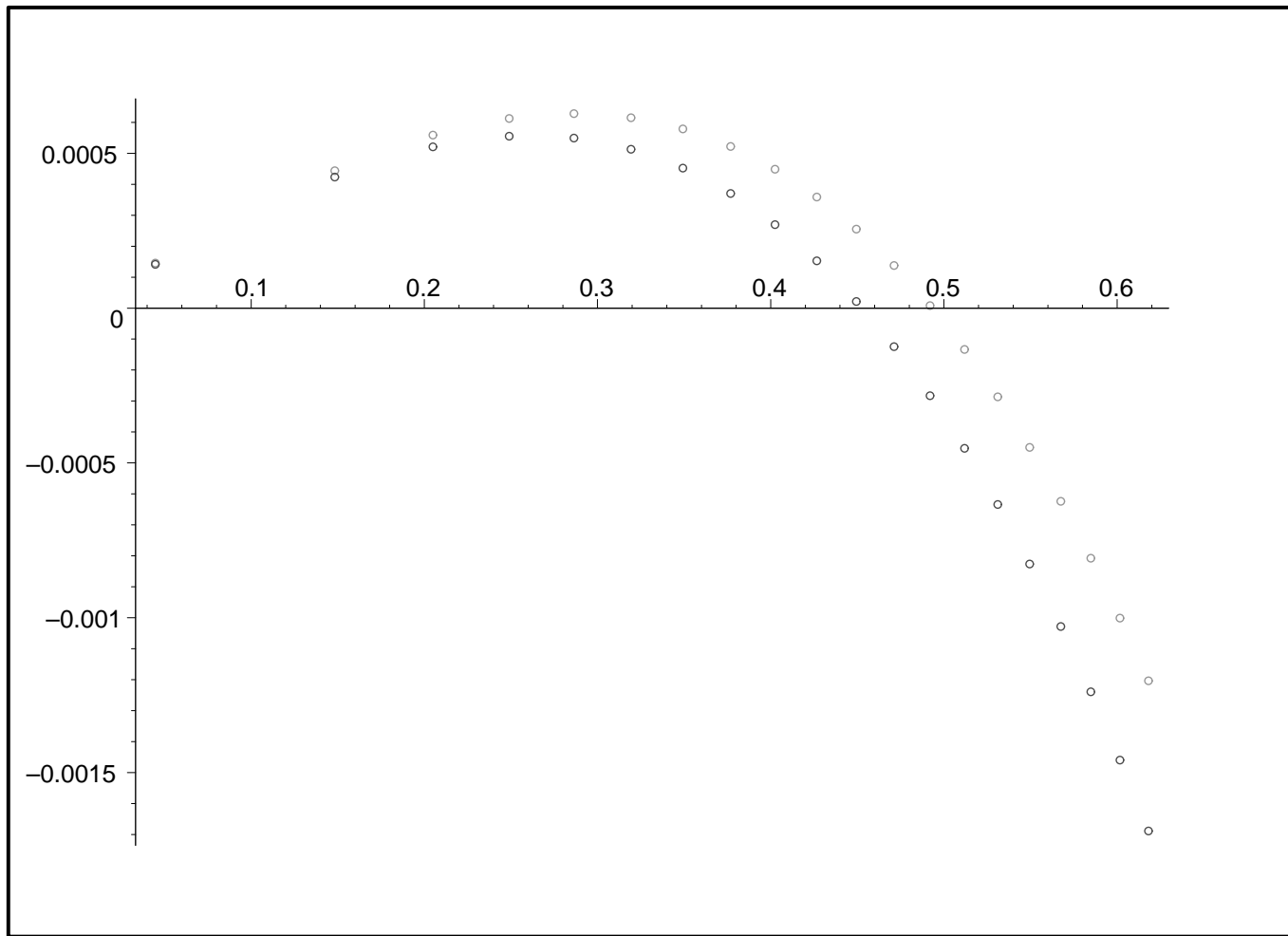


Figure 9: Growth rate  $\frac{\gamma}{\Omega}$  versus  $b = \frac{k_{\perp}}{\Omega_i} \sqrt{\frac{T_{\perp i}}{m_i}}$  from **full kinetic analysis** (at order  $(\frac{kz}{k_{\perp}})^2$ ) **[upper graph]** and the **fluid model** **[lower graph]**. ( $\frac{T_{\perp i}}{T_{\parallel i}} = 0.9$ ,  $\frac{T_{\perp e}}{T_{\parallel e}} = 0.$ ,  $T_{\parallel e} = T_{\parallel i}$ ,  $\beta_{\perp i} = 1.5$ ,  $\frac{kz}{k_{\perp}} = 0.1.$ )

Small-scale discrepancy mostly due to the expansion of plasma response function. Improvement expected when using higher order closure of the fluid hierarchy.

## Perspectives

- Benchmark the model by comparison with gyrokinetic and Vlasov-Maxwell simulations .
- Explore the nonlinear stage of parametric instabilities.
- Modelisation of **coherent structures** (magnetic holes and shocklets) observed in the **solar wind and magnetosheath**.
- Simulation of **dispersive Alfvén wave turbulence**
- Treat **electrons as a Landau fluid in hybrid simulations**.
- Explore the possible description of nonlinear Landau damping.

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