32^{nd} **EPS Plasma Physics Conference**

Tarragona, 27 June – 1 July 2005

Anisotropic wave turbulence in electron MHD

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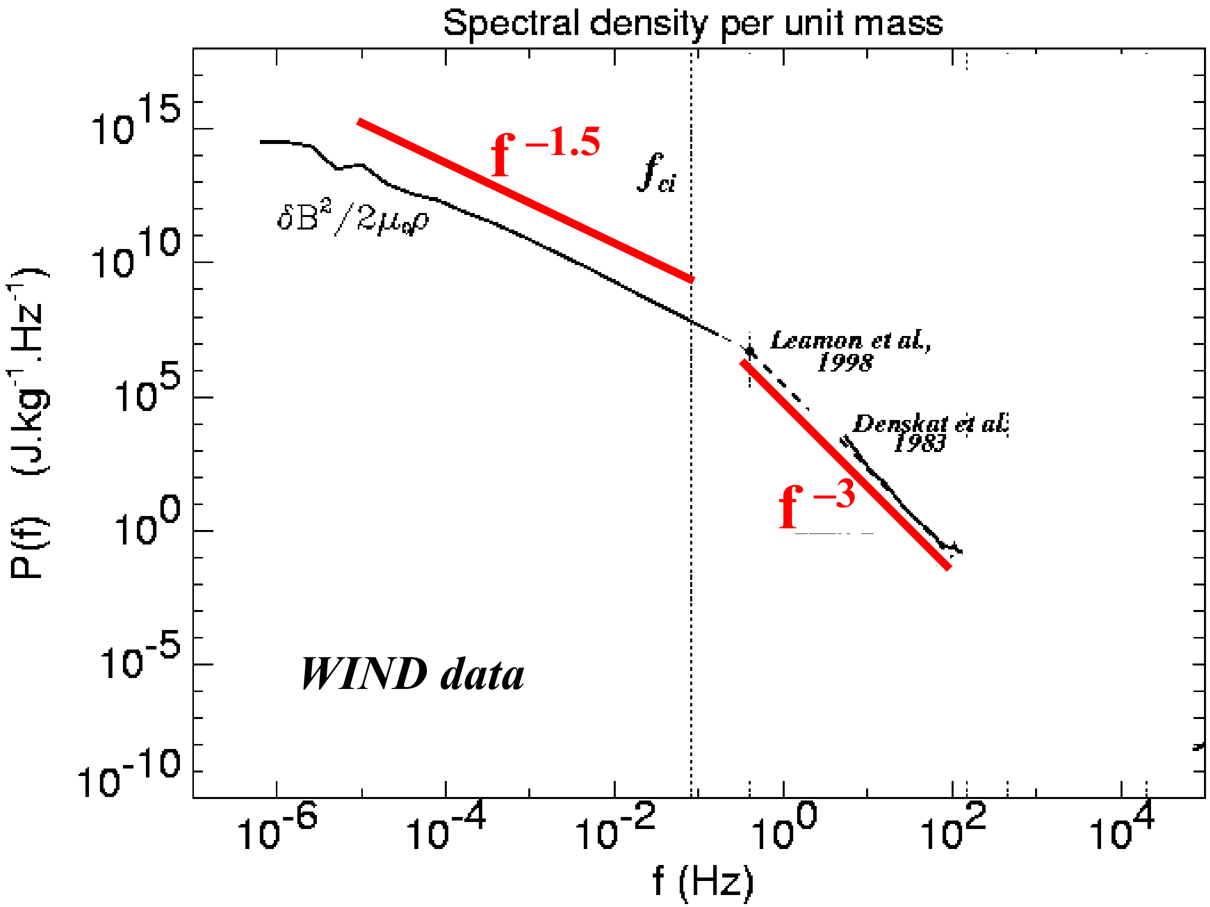
- Solar wind, small-scale turbulence and electron MHD
- Wave turbulence in electron MHD flows
- Rotating turbulence
- Conclusions and perspectives

Small-scale turbulence and EMHD

– Standard Magnetohydrodynamics (MHD) is not valid –

- Steepening of the magnetic fluctuation spectrum in the **solar wind** (from a $\mathbf{f^{-1.5} - f^{-1.8}}$ spectrum to $\mathbf{f^{-2.1} - f^{-4}}$) [see *eg*. Bale et al., 2005]



[Salem, PhD, 2000]

- Possible **signatures** of outward propagating whistler waves at small scales in the solar wind [Goldstein et al., 1994]

- Steeper power law may be attributed to **nonlinear** processes rather than dissipation [Ghosh et al., 1996; Stawicki et al., 2001]

- Small-scale magnetic **reconnection;** neutron star crusts...

→ **It is important to study formally small-scale turbulence**

(Hall MHD, electron MHD)

Small-scale electron MHD turbulence

- **What is electron MHD ?**

- Small-scale description: $L < d_i$ and $\omega_{ci} < \omega < \omega_{ce}$

- Dynamics is only given by **electrons**: ions are fixed

$$\begin{cases} \partial_t \vec{B} + \nabla \times [(\nabla \times \vec{B}) \times \vec{B}] = 0 \\ \nabla . \vec{B} = 0 \end{cases}$$

[Kingsep et al., 1990]

- Inviscid invariants: **energy (E)** and **magnetic helicity (H)**

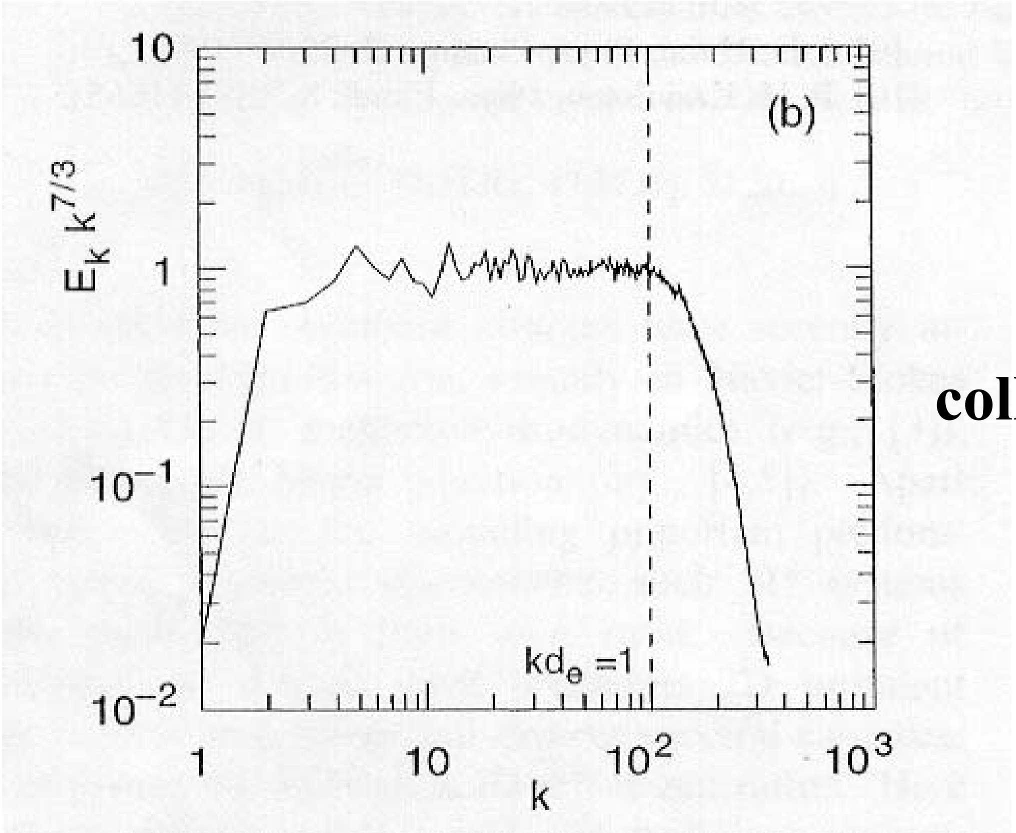
- Linear solutions = **whistler waves** (transverse circularly polarized)

$$\omega_k \sim k_{//}\, k \quad \text{with} \quad \vec{B}_0 = B_0 \vec{e}_{//}$$

- **What do we know about turbulent EMHD flows ?**

- EMHD turbulence $\neq$ MHD turbulence

- 2D DNS, isotropic (**B_o=0**), shows: **$E(k) \sim k^{-7/3}$** [Biskamp et al., 1996]



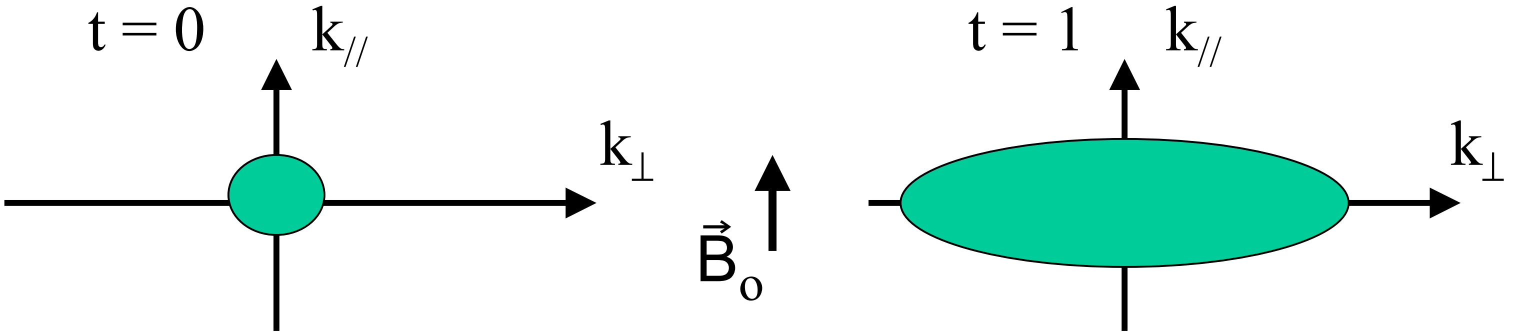
1024 x 1024 collocation points

- **What do we know about turbulent EMHD flows ?**

- Scaling $\mathbf{k^{-7/3}}$ in agreement with a K41 heuristic description

→ **Is there a whistler effect ?**

- 2D DNS, anisotropic ($\mathbf{B_o>0}$), reveals a « whistlerisation »:



[Dastgeer et al., 2000]

Wave *versus* strong EMHD turbulence

$$\begin{cases} \tau_{nl} \sim \text{length / velocity : nonlinear time} \\ \tau_{w} \sim 1/\omega_k \sim 1/k_{//}kB_o \text{ : whistler wave time} \end{cases}$$

Weak wave turbulence:

strong B_o

$\tau_{nl} \gg \tau_w$

$\tau_{tr} = \tau_{nl}\,(\tau_{nl}/\tau_w)$

anisotropy: $k_\perp \gg k_{//}$

$E(k_\perp, k_{//}) \sim k_\perp^{-5/2}\, k_{//}^{-1/2}$

Strong turbulence:

B_o weak or null

$\tau_{tr} = \tau_{nl}$

$E(k) \sim k^{-7/3}$

Whistler wave turbulence in EMHD

- We introduce: $\vec{B}(\vec{x}) = \vec{B}_o + \varepsilon \vec{b}(\vec{x}) = \vec{e}_{//} + \varepsilon \vec{b}(\vec{x})$, $\varepsilon \ll 1$
- Fourier transform the EMHD equations
- Use a complexe helicity decomposition (whistler waves are helical)

$$\vec{h}^s(\vec{k}) = \vec{h}^s{}_k = (\vec{e}_k \times \vec{e}_{//}) \times \vec{e}_k + i\, s(\vec{e}_k \times \vec{e}_{//})$$

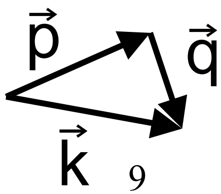
$$s = \pm \text{ (wave polarity)}, \; \vec{k} \,.\, \vec{h}^s{}_k = 0, \; i s \vec{e}_k \times \vec{h}^s{}_k = \vec{h}^s{}_k, \; i^2 = -1$$

[Craya, 1958; Kraichnan, 1973; Cambon et al., 1989; Turner, 2000]

⬇

$$\partial_t\, a^s(\vec{k}) = \varepsilon \Sigma_{sp,sq} \int L^{s,\,sp,\,sq}_{-\vec{k},\,\vec{p},\,\vec{q}}\; a^{sp}_{\vec{p}}\; a^{sq}_{\vec{q}} \exp[i(-s\omega_k + s_p\omega_p + s_q\omega_q)t]\; \underbrace{\delta_{\vec{k},\vec{p}\vec{q}}}\, d\vec{p}\, d\vec{q}$$

With: $\vec{b}(\vec{k}) = \Sigma_s\, a^s_{\vec{k}} \exp(-i\, s\, \omega_k\, t)\, \vec{h}^s{}_k$



To evaluate $\mathbf{L}^{s,sp,sq}_{\vec{k},\vec{p},\vec{q}}$, we introduce the orthonormal basis vector :

$$\vec{O}^{(1)}(\vec{p}) = \vec{n} \times \vec{e}_p\,,\quad \vec{O}^{(2)}(\vec{p}) = \vec{n}\,,\quad \vec{O}^{(3)}(\vec{p}) = -\,\vec{e}_p$$

$\vec{n} \perp$ to the triangle $\vec{k} = \vec{p} + \vec{q}$ and $\vec{n} = (\vec{k} \times \vec{p}) / |\vec{k} \times \vec{p}| = \ldots$

$$\partial_t c^s(\vec{k}) = \partial_t a^s(\vec{k})\, k_\perp / k =$$

$$\varepsilon \sum_{sp,sq} \int (s_q q - s_p p)\; M^{s,sp,sq}_{-\vec{k},\vec{p},\vec{q}}\; c^{sp}_{\vec{p}}\; c^{sq}_{\vec{q}}\, \exp[i(-s\omega_k + s_p\omega_p + s_q\omega_q)t]\; \delta_{\vec{k},\vec{p}\vec{q}}\, d\vec{p}\, d\vec{q}$$

where $\mathbf{M}^{s,sp,sq}_{\vec{k},\vec{p},\vec{q}}$ has a polar form [Turner, 2000]

The matrix **M** **has all symmetries** you need to do wave turbulence

- **Resonance condition:**

$$\begin{cases} \vec{k}+\vec{p}+\vec{q}=\vec{0} \\ s\omega_k + s_p\omega_p + s_q\omega_q = 0 \end{cases} \;\; => \;\; \frac{s_p p - sk}{q_{//}} = \frac{s_q q - s_p p}{k_{//}} = \frac{sk - s_q q}{p_{//}}$$

- **The spectral density:** $< c^s(\vec{k})\, c^{s'}(\vec{k}') > \; = q^s(\vec{k})\, \delta(\vec{k}+\vec{k}')\, \delta(s-s')$

$<\ldots>$ = ensemble average

Hypothesis: homogeneous turbulence

Kinematics: fast decorrelation for opposite travelling waves

- **Formalism:** derivation of KE, ie write $\partial_t\, q^s(\vec{k})=\ldots$

→ **Eulerian** wave turbulence approach [Newell et al, '01; Zakharov et al, '92]

- systematic asymptotic expansion in power of small nonlinearities
- write successively: $\partial_t < c^s_k\, c^{s'}_{k'} > = \ldots,\;\; \partial_t < c^s_k\, c^{s'}_{k'}, c^{s''}_{k''} > = \ldots$

…and substitute the latter (integrated in time) in the former…

At large times ($t \approx \tau_{tr} \gg \tau_w$), only resonant terms will survive

Asymptotic closure for Wave Kinetic Equations !

WHISTLER WAVE KINETIC EQUATIONS

(3-wave interactions)

[S.G. & Bhattacharjee, Phys. Plasmas, 2003]

→ **Dynamical description for energy and helicity spectra**

Kinetic equation for energy ($H(\vec{k})=0$)

$$\partial_t E(\vec{k}) = \varepsilon^2 \sum_{s,s_p,s_q} \int G\, E(\vec{q})\, (E(\vec{k}) - E(\vec{p}))\, \delta(\vec{k}+\vec{p}+\vec{q})\, \delta(g_{kpq})\, d\vec{p}\, d\vec{q}$$

where $\begin{cases} g_{kpq} = s\omega_k + s_p\omega_p + s_q\omega_q \\ G \text{ is a geometrical coefficient} \end{cases}$

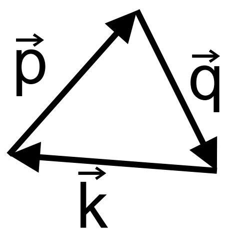
- **Properties of whistler turbulence:**

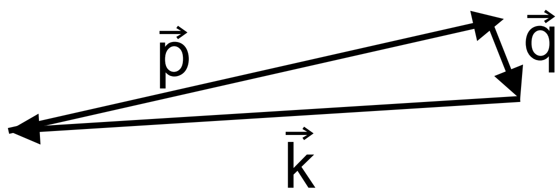
1) Detailed **conservation** of energy and magnetic helicity

2) No coupling if $p = q$ and $s_p = s_q$: a **strongly** helical perturbation **localized** in k leads to a **weak** nonlinear transfer

[Kraichnan, 1973; Waleffe, 1992; Turner, 2000]

3) In the limit of strongly **local** or **non-local** interactions, the nonlinear transfer is **essentially** $\perp \vec{B}_o$ [Dastgeer et al., 2000]





4) We can look at the limit $k_\perp \gg k_{//}$; for axisymmetric turbulence

$$\partial_t E(k_\perp,k_{//}) =$$

$$\varepsilon^2 \sum_{s,sp,sq} \int \bar{G}\, E(q_\perp,q_{//}) \left(p_\perp E(k_\perp,k_{//}) - k_\perp E(p_\perp,p_{//})\right) \delta_k \delta_\omega \, dp_\perp dp_{//} dq_\perp dq_{//}$$

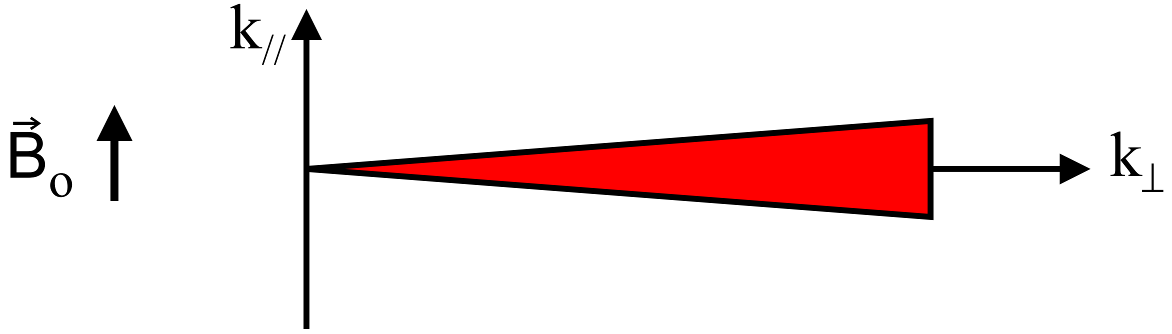
5) We find the exact power law solutions

$$E\,(k_\perp,k_{//}) \sim k_\perp^{-5/2}\, k_{//}^{-1/2}$$ (Kuznetsov-Zakharov)

→ In agreement with a simple anisotropic heuristic description

6) Direct energy cascade : positive flux

7) Domain of validity ($\tau_{tr} \gg \tau_w$): $k_{//} \gg \varepsilon^{4/3} k_\perp^{1/3}$



8) 3D modes, $k_{//}>0$, evolve **independently** to the 2D state, ie $E(k_\perp,0)$

[Cambon et al., 1989; Bartello, 1995]

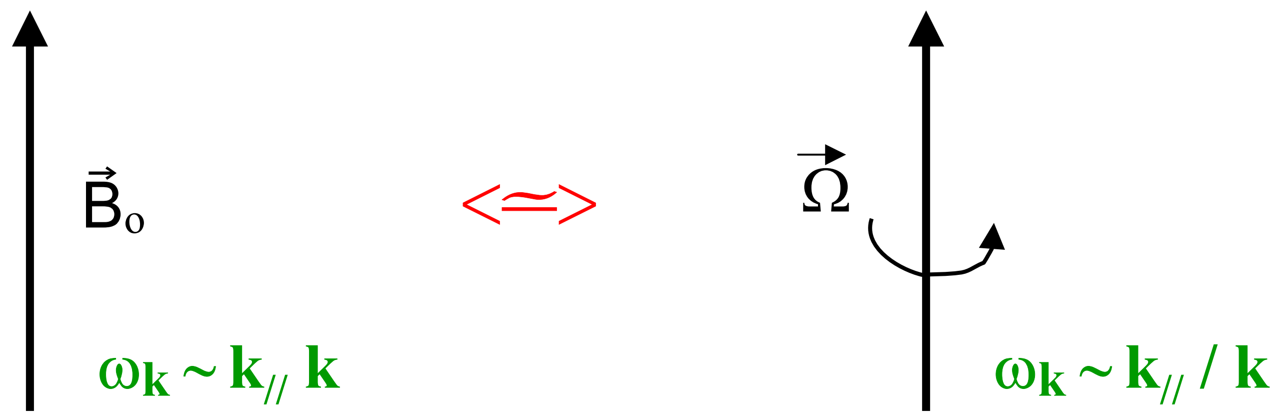
Rotating Navier-Stokes turbulence

$$\partial_t \vec{w} - 2(\vec{\Omega}\,.\nabla)\,\vec{v} = \vec{w}\,.\nabla\vec{v} - \vec{v}\,.\nabla\vec{w} + \nu \Delta \vec{w}$$

$$\vec{w} = \nabla \times \vec{v} \quad \text{and} \quad \nabla\,.\,\vec{v} = 0$$

Very strong analogy between wave turbulence in EMHD and in Navier-Stokes fluids under rapid rotation

[S.G., 2003; Morize et al., 2005]



Whistler waves

Inertial waves

Helical waves

Conclusions and perspectives

- Development of a **whistler wave turbulence** theory for EMHD

- **Strong** analogy with rapidly rotating turbulent flows

- **Steepening** of the spectrum from incompressible MHD to EMHD

  $\mathbf{E\,(k_\perp, k_{//}) \sim k_\perp^{-2} f(k_{//})}$ [S.G. et al., 2000] → $\mathbf{E\,(k_\perp, k_{//}) \sim k_\perp^{-2.5}\, k_{//}^{-1/2}}$

- Comparison with solar wind data **not** direct

  → We need simulations and more precise data (CLUSTER ?)

- Extension of this work to **Hall MHD...**