

# Role of anomalous transport in onset and evolution of neoclassical tearing modes

Sergey Konovalov<sup>1,2</sup>, Anatolii Mikhailovski<sup>2</sup>, Takahisa Ozeki<sup>1</sup>,  
Tomonori Takizuka<sup>1</sup>, Maxim Shirokov<sup>2</sup>, Nobuhiko Hayashi<sup>2</sup>

<sup>1</sup>*Naka Fusion Research Establishment, Japan Atomic Energy Research Institute*

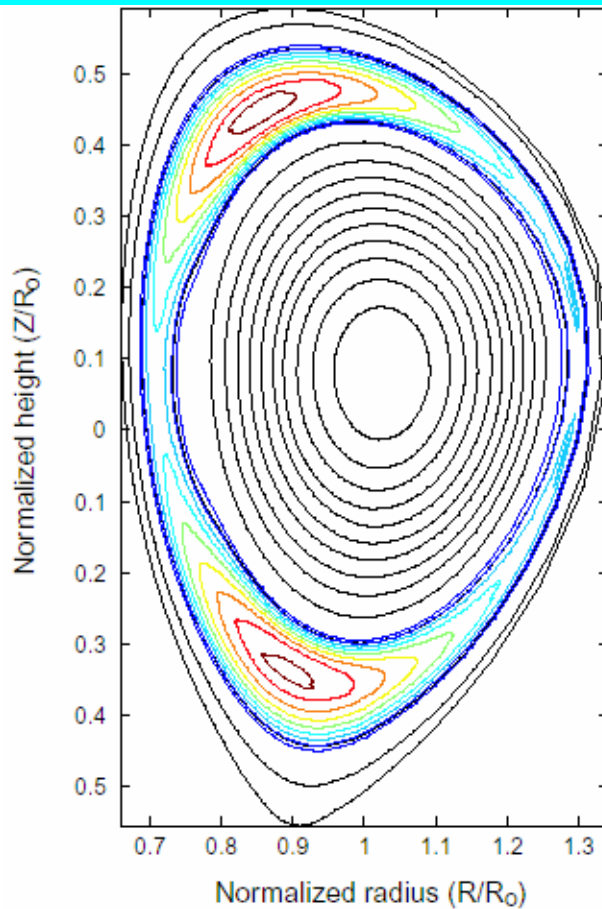
<sup>2</sup>*Institute of Nuclear Fusion, RRC “Kurchatov Institute”, Moscow, Russia*

## OUTLINE:

- What is the Neoclassical Tearing Mode
- Mechanisms providing  $\beta$ -threshold of NTM
- Cross island heat transport influence on bootstrap drive and magnetic curvature effect.
- Correction to transport threshold due to ion perpendicular viscosity effect on the bootstrap drive of NTM
- Transport modifications to the polarization current effect
- Summary and discussion



## Why NTMs are of concern



**NTMs are one of principal obstacles in attaining reactor relevant regimes**

➤ appear well below ideal limit  $\beta_{N\_NTM} \ll \beta_{N\_ideal}$

➤ degrade confinement (soft limit)

➤ limit ultimate  $\beta$   $\rightarrow P_{fusion} \sim \beta^2 B^4$

(shrink operation domain & rise \$, overall  $\downarrow$ )

➤ at worst result in disruption (hard limit)

**NTMs should be AVOIDED or SUPPRESSED**



## Evolution of the magnetic island width – 1. classical Rutherford equation

perturbation,  $\tilde{\psi} \cos \xi$ ,  $\xi = m\theta - n\zeta - \omega t$   
forms magnetic island  $W = 4(q_s R \tilde{\psi} / B_o s)^{1/2}$

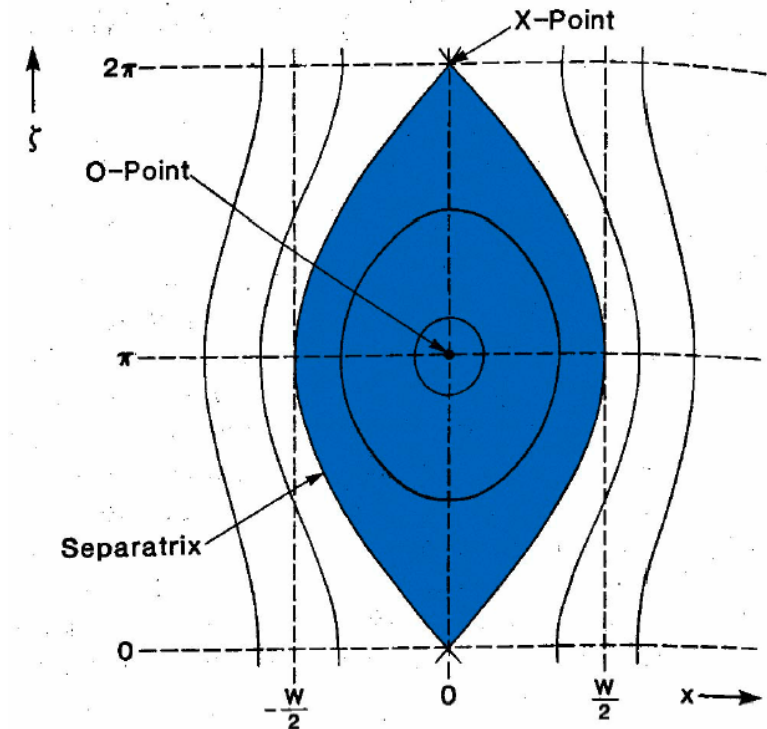
island flux surface label  $\Omega = 8x^2 / W^2 - \cos \xi$   
inside the island  $-1 < \Omega < 1$

Ampere law  $\Delta' \tilde{\psi} = 2\mu_0 R \int_{-\infty}^{\infty} dx \int d\xi J_{\parallel} \cos \xi$

Rutherford-1973 :

induced current  $J_{\parallel} = J_{\parallel}(\Omega) = \frac{1}{\eta} \frac{\partial \tilde{\psi}}{\partial t} \langle \cos \xi \rangle$

Island width evolution: (Rutherford equation)



$$0.41 \tau_r \frac{dW}{dt} = r_s^2 \Delta'$$

For fast parallel transport the **pressure is flattened inside the island**

## Evolution of the neoclassical island – 2. modified Rutherford equation

Bootstrap is removed from inside the island,  $J_{bs} \sim \frac{dp}{dr}$  whilst remains outside.

$$J_{\parallel}(\Omega) = \frac{1}{\eta} \frac{\partial \tilde{\psi}}{\partial t} \langle \cos \xi \rangle + \frac{1}{\eta n e B} \langle \mathbf{B} \cdot \nabla \cdot \pi_{\parallel e} \rangle = \text{Inductive} + \text{Bootstrap}$$

Particle drift across the magnetic surfaces provides **curvature** and **polarization** drift contributions into NTM dynamics

$$\mathbf{B} \cdot \nabla \frac{J_{\parallel}}{B} = -\nabla \cdot \frac{\text{Curvature } \mathbf{B} \times \nabla p}{B^2} - \nabla \cdot \frac{\text{Polarization drift } \mathbf{B}}{B^2} \times \left( \rho \frac{d\mathbf{V}}{dt} + \nabla \cdot \boldsymbol{\pi} \right)$$

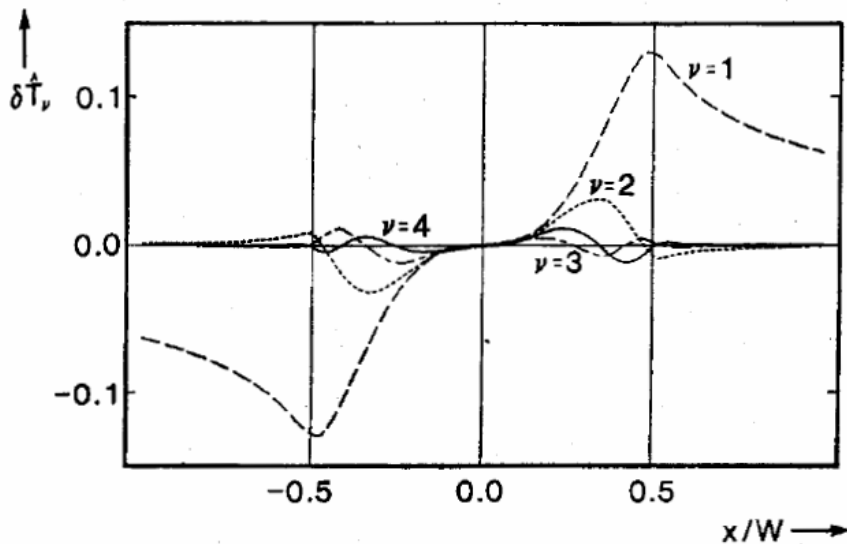
**All new contributions** are sensitive to transport phenomena:

$$\left( \tilde{T}_{e,i}(r), \tilde{n}_e(r), \tilde{\mathbf{V}}(r) \text{ to be determined} \right)$$

## Temperature perturbations associated with magnetic islands -1

First analysis for finite parallel thermal conductivity Fitzpatrick-1995 (MHD-Braginskii) and Gorelenkov et al -1996 (Drift kinetic)

Heat balance is  $\nabla \cdot \mathbf{q} = \chi_{\parallel} \nabla_{\parallel}^2 T + \chi_{\perp} \nabla_{\perp}^2 T = 0$   $W_c \sim (\chi_{\perp} / \chi_{\parallel})^{1/4}$



(Fitzpatrick - 95)

- $W \gg W_c$  - temperature is **flattened**

$$T = T(\Omega = 1);$$

$$\delta T(x, \xi) = \sum_{\nu} T_{\nu}(x) \cos \nu \xi;$$

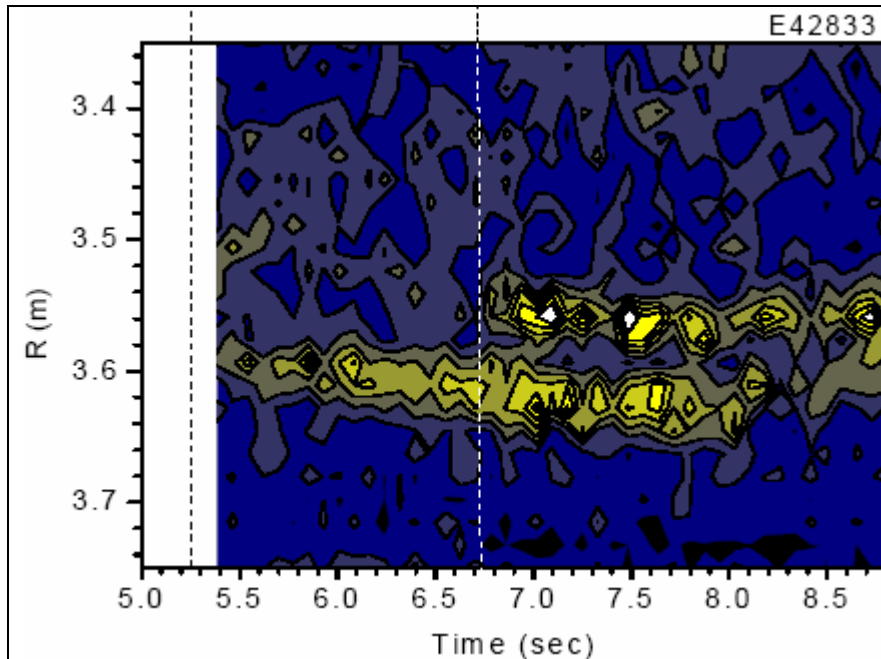
$$\Delta_{bs} \sim \frac{1}{W}$$

- $W \leq W_c$  - there is **no flattening**,

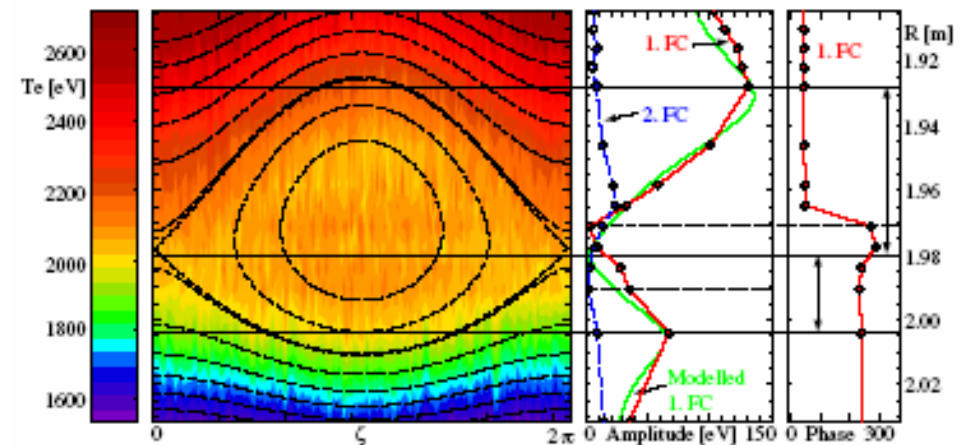
$$T = f(x) \cos \xi;$$

$$\Delta_{bs} \sim \frac{W}{W_c^2}$$

## Temperature perturbations associated with magnetic islands -2



NTM evolution from ECE measurements in JT-60U (Nagasaki et al. 2004)



Simulated and ECE detected structure of  $\tilde{T}_e(r)$  in AUG (Meskat et al 2001)

**Fitzpatrick model vs. experiment:**

1)  $\chi_{\perp}$  should be anomalously high;

2) heat flux limit for  $\chi_{\parallel} = \chi_{sp} / \sqrt{1 + (3.16V_e / v_{e,i} L_c)^2}$



## Anomalous transport affects NTM threshold

$$\frac{\tau_r}{r_s} \frac{dw}{dt} = r_s \Delta' + \beta_p \left[ \underbrace{\frac{r_s C_{bs}}{w}}_{\text{Conventional tearing}} \times \frac{w^2}{w^2 + w_*^2} + \underbrace{\frac{r_s C_{mw}}{w}}_{\text{Bootstrap Drive}} \times \frac{w}{w + w_{*mw}} + \underbrace{\pm g \frac{C_{pol}}{w^3}}_{\text{Magnetic curvature}} + \underbrace{\Delta_{EC}}_{\text{Polarization drift}} \right]$$

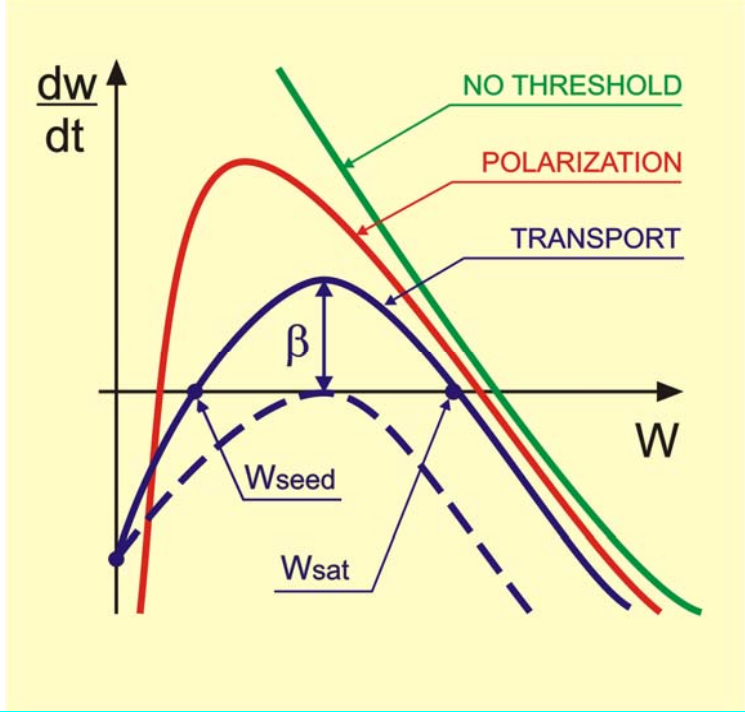
Transport threshold

$$\beta_{p, \text{marg}}^{\chi} \propto W_* \frac{\Delta'}{\sqrt{\epsilon}} \cdot \frac{L_p}{L_q}$$

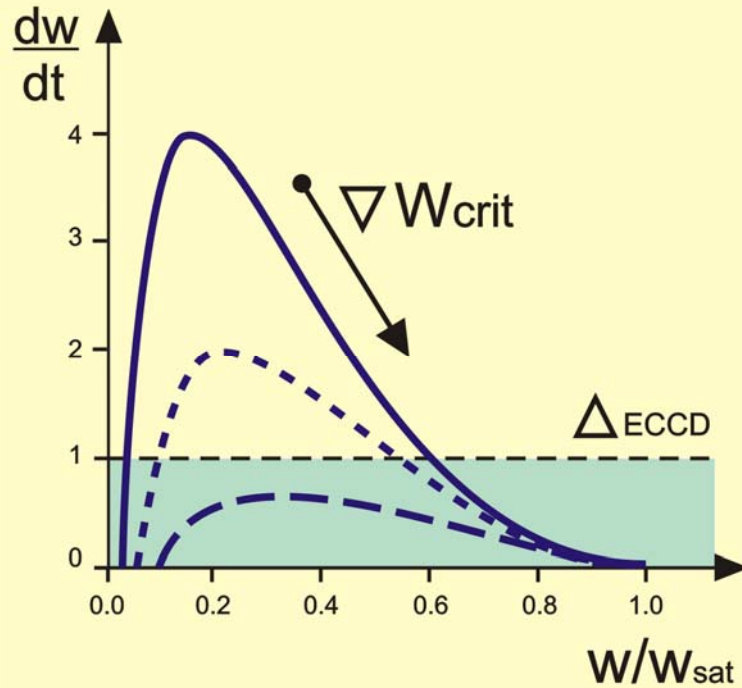
Anomalous transport

Polarization drift

$$\beta_{p, \text{marg}}^{\text{poll. curr.}} \propto -\Delta' \rho_{pi} \sqrt{g \frac{L_p}{L_q \epsilon^{3/2}}}$$



## Finite parallel heat transport effect on NTM evolution / suppression



Adding non-resonant helical perturbation rise effective value of  $\chi_{\perp eff}$  (Yu et al 2000):

$$\hat{\mathbf{b}} = \mathbf{b} + b_{ext\_r} \mathbf{x}$$

$$\nabla_{\parallel} = \mathbf{b} \cdot \nabla + b_{ext\_r} \frac{\partial}{\partial x}$$

$$\chi_{\perp} \rightarrow \chi_{\perp}^{eff} = \chi_{\perp} + \chi_{\parallel} \langle b_{ext\_r}^2 \rangle_{\theta}$$

$$B_{ext\_r} / B_0 \approx 10^{-4} \text{ suppress NTM}$$

Rise of  $W_*$  facilitate ECCD suppression

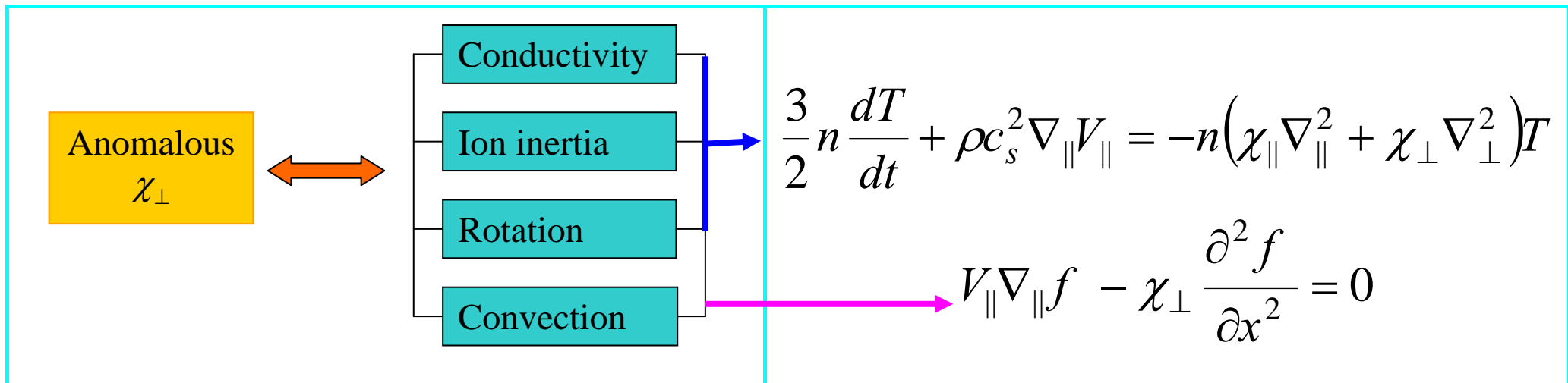
Possible way to suppress NTM by externally applied nonresonant helical perturbation

## Transport threshold model of NTM

$$\beta_{p, \text{m arg}}^{\chi} \propto W_*, \quad W_* \text{ depends on the transport model used:} \quad W_* \propto (\chi_{\perp} / \chi_{\parallel})^{1/4}$$

$$\chi_{\perp} = \chi_{\perp}^{\text{gB}} \propto T^{3/2} / B^2; \quad \chi_{\parallel} = \chi_{\text{Spitzer}} \quad \beta_{p, \text{m arg}}^{\chi} \propto \rho_{pi}^* \sqrt{v_{ii}^*}$$

**Anomalous cross-island heat transport competes with series of parallel transport mechanisms in forming perturbed temperature profile:**





## Varieties of the heat transport mechanisms

Model	Basic equations	Characteristic island width [Mikhailovskii 2005]
<b>Collisional<sup>1</sup></b> (Fitzpatrick 95)	$\nabla \cdot \mathbf{q} = 0, \quad \mathbf{q} = -n_0(\chi_{\parallel} \mathbf{b} \nabla_{\parallel} + \chi_{\perp} \nabla_{\perp})T$	$W_{col} = 2^{3/2} \left( \frac{L_s^2 \chi_{\perp}}{k_y^2 \chi_{\parallel}} \right)^{1/4}$
<b>Rotational<sup>3</sup></b> (Konovalov 2002)	$n_0 \frac{d_0 T}{dt} = -\frac{2}{3} \nabla \cdot \mathbf{q}_{\perp}$	$W_{rot} = \left( \frac{128\sqrt{2} \chi_{\perp}}{3\pi \omega} \right)^{1/2}$
<b>Inertial<sup>4</sup></b> (Mikhailovskii 2003)	$\rho_0 c_{is}^2 \nabla_{\parallel} V_{\parallel} = -\frac{2}{3} \nabla \cdot \mathbf{q}_{\perp}, \quad \rho_0 \frac{d_0 V_{\parallel}}{dt} = -n_0 \nabla_{\parallel} T$	$W_{inert} = \left( \frac{128\sqrt{2}\pi \omega L_s^2 \chi_{\perp}}{3 k_y^2 c_{is}^2} \right)^{1/4}$
<b>Convective<sup>5</sup></b> (Mikhailovskii 2003)	$\nabla_{\parallel} q_{\parallel} - \frac{3}{2} \chi_{\perp} \frac{\partial^2 p}{\partial x^2} = 0,$ $\frac{5}{M} \nabla_{\parallel} (pT) - \chi_{\perp} \frac{\partial^2 q_{\parallel}}{\partial x^2} = 0$	$W_{conv} = \left( \frac{2^{11/2} \chi_{\perp} L_s}{c_s k_y} \right)^{1/3}$

## Bootstrap drive and curvature effect for series of heat transport mechanisms

1) Partial contributions from temperature and density gradients

$$\Delta_{bs} = \sum_{A=n, T_e, T_i} a_{bs,A} \frac{W}{W^2 + W_{bs,A}^2}, \quad \Delta_{mw} = \sum_{A=n, T_e, T_i} a_{mw,A} \frac{1}{W + W_{mw,A}}.$$

(Curvature effect weakening – Lutjens et al 2001)

2) characteristic island widths  $W_{bs,A}$  and  $W_{mw,A}$  are determined by the combination of transport mechanisms

$$\frac{1}{W_{bs,A}^2} = \sum_k \frac{1}{W_{bs,A,k}^2}, \quad \frac{1}{W_{mw,A}} = \sum_k \frac{1}{W_{mw,A,k}}.$$

3) “Effective” critical island width is determined by the fastest of parallel transports



## Perturbed $T_e$ and $T_i$ are formed due to different transport mechanisms

<b>Electrons</b>		<b>Ions</b>	
<b>Convective</b>	<i>Collisional</i>	<b>Inertial</b>	<i>rotational</i>
<p><b>Convection is dominant</b></p> $W_* \propto \chi_{\perp}^{1/3}$ <p><b>Collisional</b> (heat flux limit for <math>\chi_{\parallel}</math>)</p> $W_* \propto \chi_{\perp}^{1/4}$		<p><b>Ion transport mechanisms are frequency dependent</b></p> <p>For <math>\omega \approx \omega_*</math>, and <math>W_{\min} \approx \Delta_{orbit}</math></p> <p><b>Inertial</b> for <math>W/W_{\min} &gt; \epsilon^{-1/2}</math>.</p> $W_* \propto (\omega \chi_{\perp})^{1/4}$ <p><b>Rotational</b> for <math>1 &lt; W/W_{\min} &lt; \epsilon^{-1/2}</math>.</p> $W_* \propto (\chi_{\perp} / \omega)^{1/2}$	

## Taking into account partial contributions from electron and ion temperatures

**Bootstrap drive** (high aspect ratio limit, neglecting  $\nabla n$  effect)

$$\Delta_{bs} = 3.89 \frac{\varepsilon^{1/2} r_s}{s} \left[ 0.40 \cdot \frac{\beta_{pe}}{L_{Te}} \cdot \frac{W}{(W^2 + W_{crit,e}^2)} - 0.17 \cdot \frac{\beta_{pi}}{L_{Ti}} \cdot \frac{W}{(W^2 + W_{crit,i}^2)} \right]$$

**Magnetic well (curvature) effect**

$$\Delta_{mw} = -3.16 \frac{\varepsilon^2 r_s}{s^2} \left( 1 - \frac{1}{q^2} \right) \left[ \frac{\beta_{pe}}{L_{Te} (W + W_{mw,e})} + \frac{\beta_{pi}}{L_{Ti} (W + W_{mw,i})} \right]$$

**Relative role of the magnetic well effect rises at small island limit:**

$\Delta_{bs}$  is weakened ( $\nabla T_i$  effect) and drops faster at  $W \leq W_{crit}$ ,

thus  $(-\Delta_{mw}) > \Delta_{bs}$  gives **NTM suppression**

For collisional mechanism NTM suppressed at

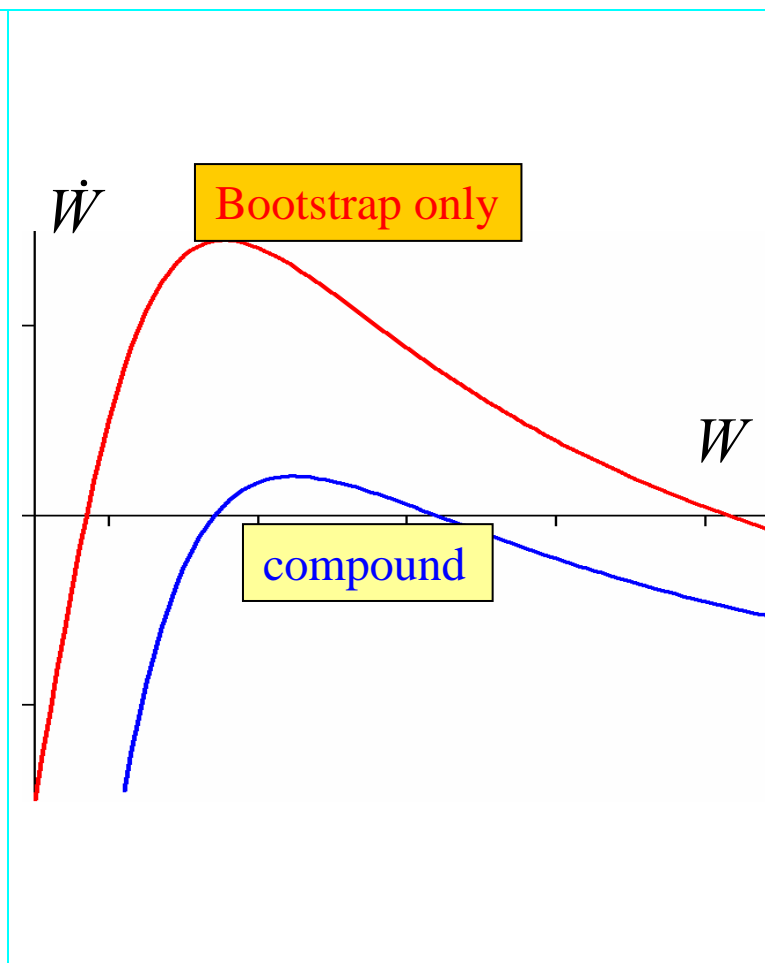
$$W < W^{comp} \approx 29 \varepsilon^{3/2} W_{col}.$$

## Generalized transport threshold model

➤ **The relative role of MW effect rises at small island limit.** (Full suppression instead of reduction at  $W \approx W_*$ )

➤ **Compound = Bootstrap+MW+transport** gives more favorable predictions for NTM stability

➤ Scaling for  $\beta_{p, \text{margin}}$  should be modified :

$$\beta_{p, \text{margin}} \neq \beta_{p, \text{margin}}(\rho_{p,i}^*, v_{ii}^*) \Rightarrow \beta_{p, \text{margin}}(\chi_{\perp, ei}, \omega, \dots)$$


## Perpendicular ion viscosity effect on the bootstrap drive of NTM

In the presence of ion perpendicular viscosity [Konovalov 2002]  $\tilde{J}_{bs} = \tilde{J}_{bs}^e + \tilde{J}_{bs}^i + \tilde{J}_{bs}^E$

Electron part  $\tilde{J}_{bs}^e$  does not change,

Ion contribution  $\tilde{J}_{bs}^i$  is reduced by  $W^2 / (W^2 + W_\mu^2)$ , where  $W_\mu = (\mu_{\perp i} / \varepsilon^{1/2} v_i)^{1/2}$

New component due to perturbed electrical field appears:

$$\tilde{J}_{bs}^E = -\frac{2.46 \varepsilon^{1/2} c e n_0 \tilde{E}_x}{B_\theta} \cdot \frac{W_\mu^2}{W^2 + W_\mu^2} \quad \Delta_{bs,E} = -3.79 \frac{\varepsilon^{1/2} r_s \beta_{pe}}{sW L_{pe}} \frac{W_\mu^2}{(W^2 + W_\mu^2)} \frac{\omega}{\omega_{pe}}$$

For islands, rotating in the ion diamagnetic drift direction viscosity can cancel threshold

## Simple estimations of the viscosity effect on bootstrap drive

**Typical initial island**  $W_{seed} \approx \rho_b < W_\mu$  :

- $\mu_\perp \approx \chi_\perp \approx \chi_{\perp g-Bohm} \approx \rho_i^2 V_{Ti} / qR$

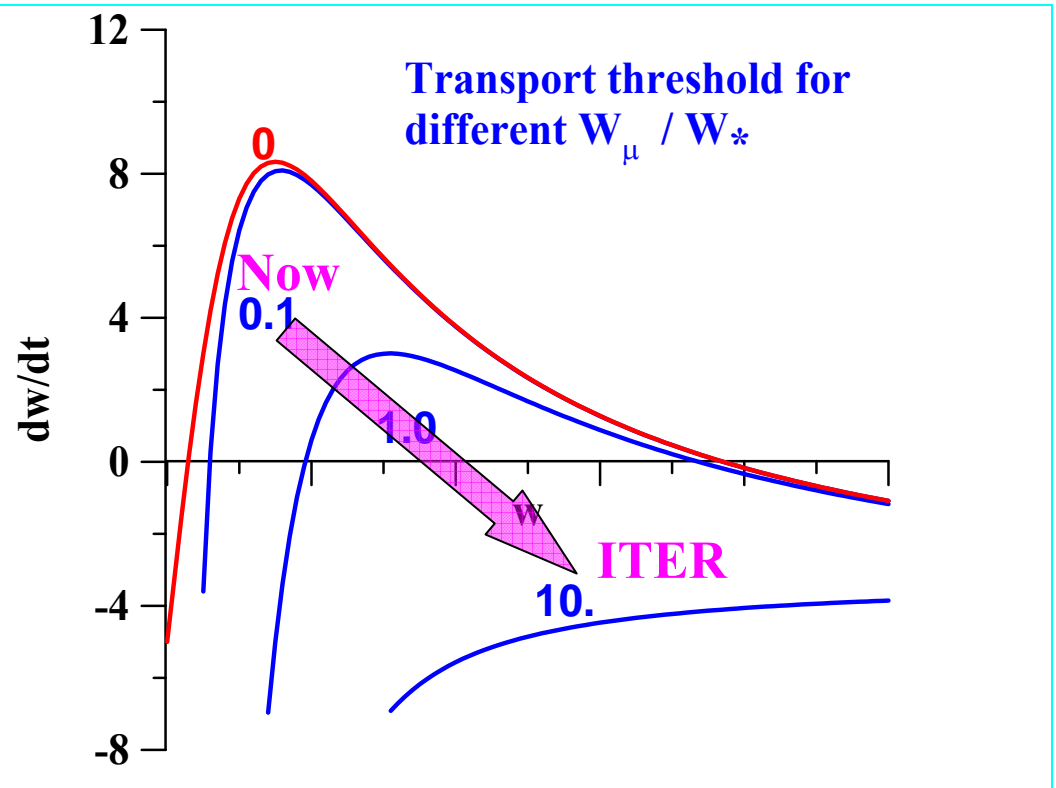
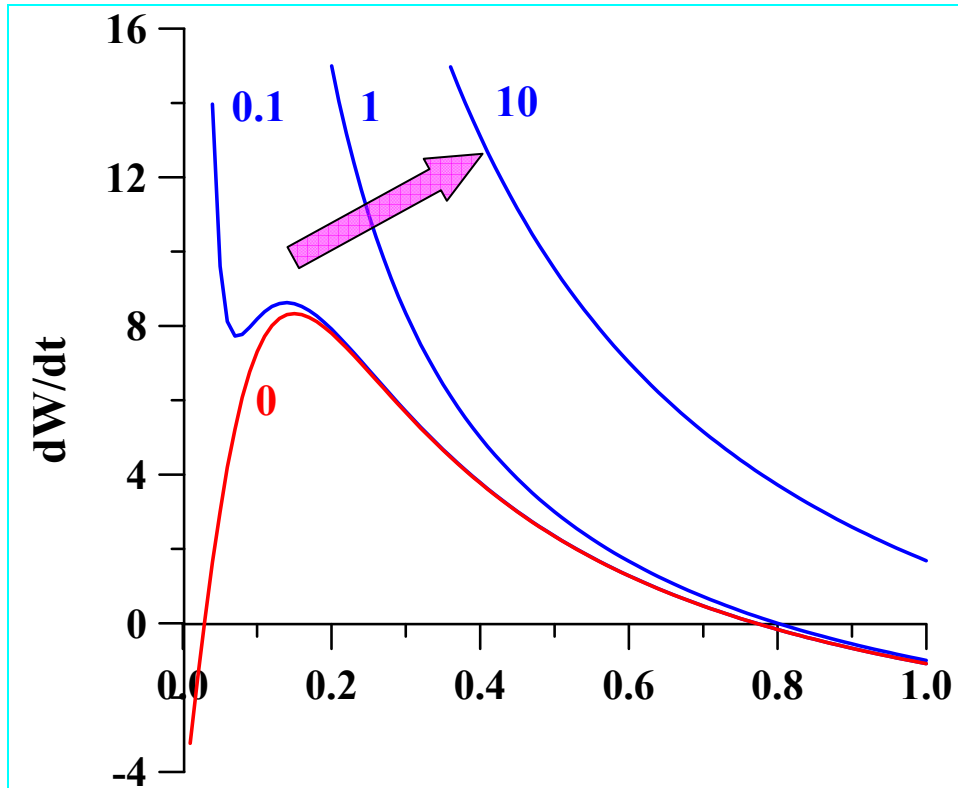
Then  $W_\mu \approx \rho_i^2 / \varepsilon^2 v_i^{*1/2}$ , where  $v_i^* \equiv v_i qR / \varepsilon^{3/2} V_{Ti} < 1$  (in banana regime)

- $W_\mu / \rho_b \approx (q \varepsilon^{3/2} v_i^{*1/2})^{-1} \gg 1$

**Perpendicular viscosity is *a priory* important**



## Relative role of viscous effect rises with plasma temperature



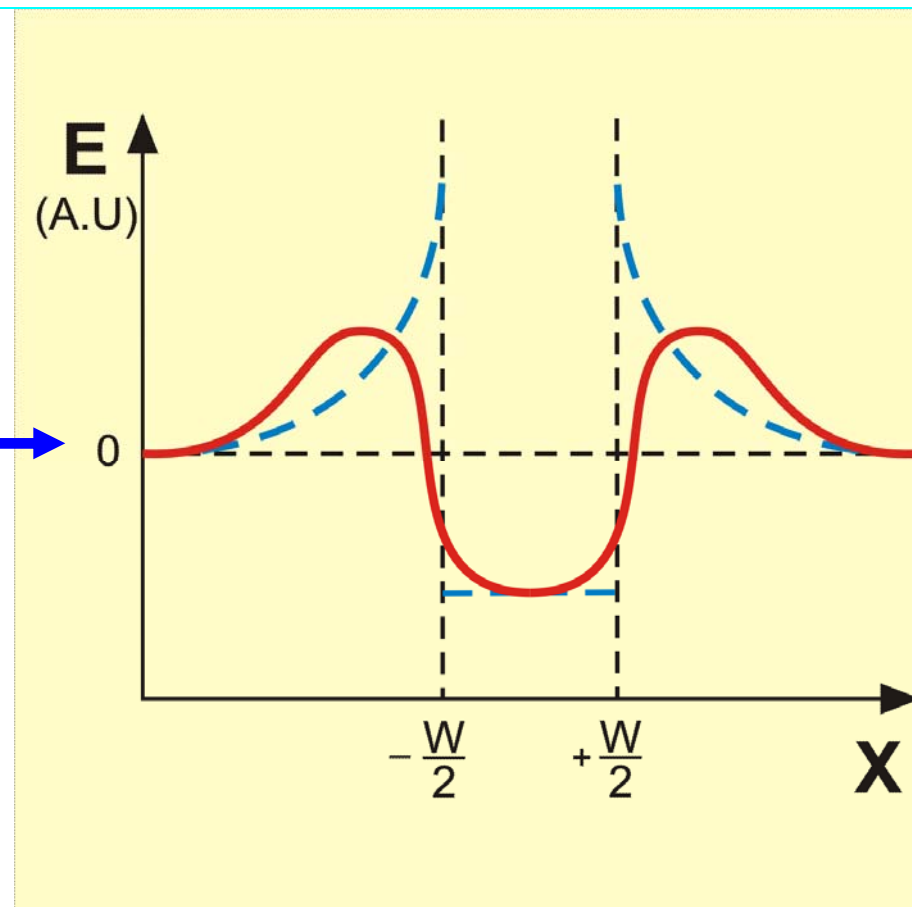
$$W_* \approx \left( \frac{\chi_\perp L_s}{V_{Te} k_y} \right)^{1/3}, \quad W_\mu / W_* \sim T^{5/3}$$

For island rotating with frequency of  $\omega = \omega_{*,e}$   
viscous bootstrap is stabilizing

## Standard polarization current threshold model can not give quantitative predictions of NTM stability

### Drawbacks:

- $\Delta_p$  changes sign with change of the island rotation direction, but always referred as stabilizing [Mikhailovskii review 2003]
- Amplitude depends on the perturbed flow velocity profile [Connor et al 2001] It drastically decreases when realistic perturbed velocity profile is considered [Konovalov 2001]  $\Delta_p = \alpha_{pol} \Delta_{pol}^{analytic}$ ,  $\alpha_{pol} \leq 0.1$



## Perpendicular viscosity modifies (removes!) collisionality dependence of the polarization threshold

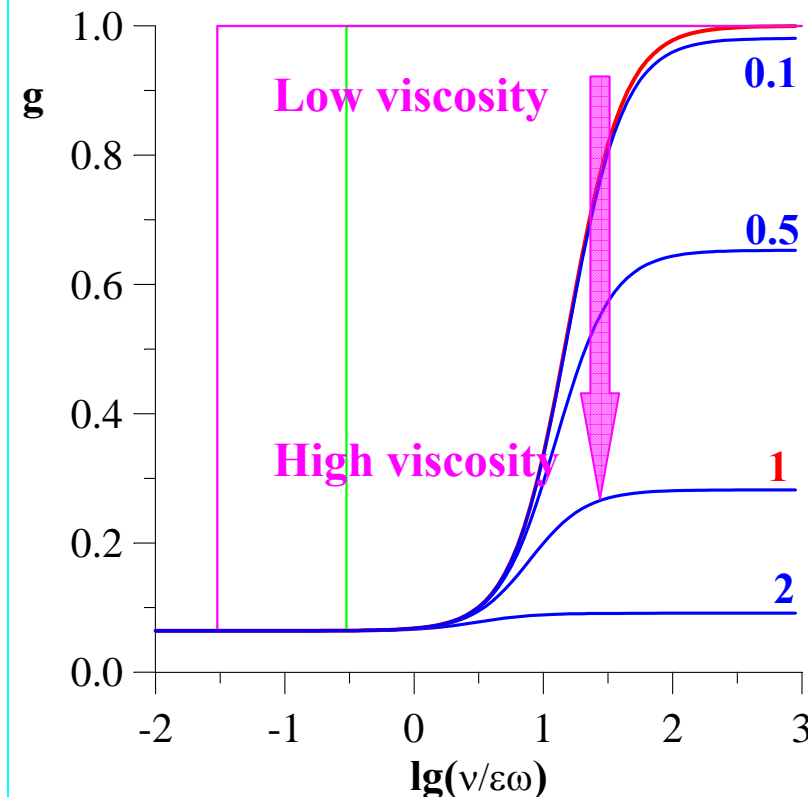
- Low collisionality limit gives smallness

$$g \rightarrow \varepsilon^{3/2}$$

Amplitude and collisionality:

$$\Delta_p = g \alpha_{pol} \Delta_{pol}^{analytic} \rightarrow 0.01 \Delta_{pol}^{analytic}$$

**Standard polarization current effect is important at extremely small (hardly detectable) island width.**



Function  $g$  for different ratios of  $W_\mu / W$

## Summary

### What we know (what can be used in NTM simulations)

- **Partial contributions from electrons are of principal importance**
  - they enter in a different way into modified Rutherford equation
  - perturbed electron and ion temperature profiles are determined by the different transport mechanisms.
  
- **Dominant mechanisms competitive to the cross-island heat**
  - parallel heat convection for electrons,
  - parallel plasma inertia for ions
  
- **In the small island limit  $\Delta_{bs} \approx \Delta_{mw}$  generalized transport threshold model**
  - gives more favorable predictions for NTM stability
  - modifies scaling law for  $\beta_{\text{onset}}$ .

## Discussion - 1

### What we understand (what to be included in the next step NTM modeling)

- Equation for island rotation frequency is necessary
- Ion transport and perpendicular viscosity provide  $\beta_{onset}(\rho_*, \nu) \rightarrow \beta_{onset}(\rho_*, \nu, \omega, \dots)$
- Viscous contribution to bootstrap drive in the small island limit
  - Can be of the order of “standard” bootstrap drive  $\Delta_{bs,E} \approx \Delta_{bs}$
  - destabilizing for the island rotating in the ion diamagnetic drift direction
  - can cancel out transport threshold of the NTM onset.
- Polarization current effect seems to be too weak, but we still need it or an alternative effects (Smolyakov-Lazzaro 2004) to compensate viscous bootstrap

## Discussion - 2

### On the standard transport threshold model:

**Electrons:** Establishment of  $\tilde{T}_e(r)$  and  $\tilde{J}_{bs,e}$  contribution to the NTM evolution are more or less understood

**Ions:** Finite orbit width effect [Poli et al 2002]

$$\tilde{J}_{bs,i} \rightarrow 0 \text{ for } W \geq \Delta_{banana}$$

**Density:**  $W_{d,n} \sim 4W_{d,e}$  [Fitzpatrick – 95], i.e.  $\Delta_{bs,n}$  is strongly suppressed.

Further analysis is necessary: particle transport should be considered with strict requirement of quasineutrality, sound wave dominated parallel transport etc..

**Local values** of  $\chi_{\perp}$ ,  $\mu_{\perp}$ . Selfconsistent modification of

- heat transport [Connor 2001, Zabiego 2002],
- viscosity and island rotation [Shaing – 2004]

## Conclusions

### ➤ Theory needs further development

There must be much more to the theoretical understanding of the NTM onset and evolution, where transport phenomena play the key role.

### ➤ Experimental study is far ahead at the present

- NTM avoidance by plasma current profile (JT-60U, JET, AUG, DIII-D, ...) and seed island size (JET, AUG, ...) controls
- Suppression (JT-60U, JET, AUG, DIII-D, ...) or avoidance with use of preventive ECCD (JT-60U)
- High performance regimes with moderate NTM (FIR regime in AUG)

**Overall, experiments guided by semi-qualitative theory show optimistic perspectives for reactor relevant plasmas.**

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